April 3, 2020

To: Families and Caregivers of CMSD Students:

CMSD continues to send regular updates on the services and supports we are providing during the unprecedented closure of schools, as part of a state-wide effort to contain the spread of the COVID-19 virus.

In addition to the grab-and-go meals we are providing at 22 school sites each day, CMSD is also distributing learning packets, and I want to personally emphasize the value of these academic enrichment materials that are handed out with meals and posted on the CMSD website: ClevelandMetroSchools.org.

Research shows that children learn best when learning is continuous, which is why CMSD educators are working hard to produce interesting and thought-provoking materials that will keep students engaged and that will keep their minds active during this long break from school.

Recognizing that students are used to a consistent school schedule, I strongly encourage you to work with your child to develop a routine at home, to make time and space for quiet reading and active engagement with their learning materials and to praise them for their attention to their studies and their personal growth.

CMSD’s Academic Enrichment Plan, posted on CMSD’s website, includes lessons and a recommended daily schedule for students at every grade level, from PreK to 12. Digital lessons can be accessed online and print materials are available for pickup at all meal sites.

Thank you for the opportunity to emphasize the importance of academic enrichment in our students’ experience during this unprecedented time away from school. And thank you for the important role you play every day in our shared commitment to the safety, growth and future of Cleveland’s children.

Thank you.

Eric S. Gordon
CEO
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<th>April 6</th>
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<td><strong>ELA I_II</strong>&lt;br&gt;Data Plots Article: Determine central idea and provide summary of a text.</td>
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<td><strong>ELA III_IV</strong>&lt;br&gt;Smartphones Put Your Privacy at Risk: Read and answer questions</td>
<td><strong>ELA III_IV</strong>&lt;br&gt;Smartphones Put Your Privacy at Risk: Read and answer questions</td>
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<td><strong>ELA III_IV</strong>&lt;br&gt;What Adolescents Miss When We Let Them Grow Up in Cyberspace: Read and answer questions</td>
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<td><strong>Chemistry</strong>&lt;br&gt;Standards of Measurement: Read and answer questions</td>
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<td>World History WWI: Read and answer questions Holocaust Videos: Watch and answer questions</td>
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<td>US History Political Parties: Read and answer questions</td>
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<td>World History WWI: Read and answer questions</td>
<td>World History New Deal: Read and answer questions Writing Assignment</td>
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<td>US History Manzan Camp: Read and answer questions Photos Japanese in America: Read and answer questions</td>
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### Suggested Daily Schedule: Grades 9 - 12

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00 – 9:00 am</td>
<td>Wake up, make your bed, eat breakfast and get ready for an awesome day!</td>
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<tr>
<td>9:00 – 9:40 am</td>
<td>Mathematics</td>
</tr>
<tr>
<td>9:40 – 10:20 am</td>
<td>English Language Arts</td>
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<tr>
<td>10:20 – 11:00 am</td>
<td>Science</td>
</tr>
<tr>
<td>11:00 – 12:30</td>
<td>Lunch, World Languages, and Free Time</td>
</tr>
<tr>
<td>12:30 – 1:10 pm</td>
<td>Social Studies</td>
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<td>1:10 – 1:40</td>
<td>Afternoon Exercise</td>
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<tr>
<td>1:40 – 2:10</td>
<td>Current Events – watch the news or read the newspaper OR Language Acquisition</td>
</tr>
<tr>
<td>2:10-2:30</td>
<td>Social-Emotional Learning/Reflection/Organize for the Next Day</td>
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</tbody>
</table>

### Family Suggestions

#### Parent Suggestions
- How can I support my student as a learner outside of school?
  - Familiarize yourself with your child’s learning calendar.
  - Encourage your child to do their best when completing tasks and assignments.
  - Contact your child’s teacher or the district’s homework hotline when you or your child have questions or need feedback.
  - Support your child in starting the daily work early in the day. Waiting until the late afternoon or evening to start work adds unnecessary stress and creates missed opportunities for collaboration and feedback.
  - Remind your child to take frequent breaks to stay focused.
  - Consider designating a dedicated workspace to maximize time on task and facilitate learning.

#### Student Suggestions
- How can I continue learning outside of school?
  - Complete work on your suggested learning calendar.
  - Put in your best effort when completing tasks and assignments.
  - Contact your teacher when you need help. Teachers are available via e-mail, your school’s online learning program or on the district’s homework hotline.
  - Let your teacher know if you have access to a phone or computer.

- How can I stay organized?
  - Start your work early. Waiting until the late afternoon or evening to start work adds unnecessary stress and creates missed opportunities for collaboration and feedback.
  - Take short breaks to increase focus and stay motivated to complete tasks on time.
  - Find a quiet place to complete your work.
## Additional Student Supports

<table>
<thead>
<tr>
<th>Individual Supports</th>
<th>See “Individualizing Support for Students” for more information on how to provide additional support to your child while at home.</th>
</tr>
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</table>
| **English Language Learners** | **Enrichment Packet**  
- Daily language learning is important! The following links/resources are available for students to access daily language learning.  
- ¡El aprendizaje diario de idiomas es importante! Los siguientes enlaces/recursos están disponibles para que los estudiantes accedan al aprendizaje diario de idiomas.  
- Kujiifunza lugha ya kila siku ni muhimu! Viungo vifuatavyo/rasilimali vinapatikana kwa wanafunzi kupata mafunzo ya lugha ya kila siku.  
- दैनिक भाषासिक्ि महत्त्वपूर्ण छ! तलका लिंकहरू / स्रोतहरू विद्यार्थीहरूको लागि दैनिक भाषा सिक्िे पहुँचको लागि उपलब्ध छन्।  
- |
| **AP** | College Board is offering free online courses on YouTube! Follow the link below to access their information.  
[https://apstudents.collegeboard.org/coronavirus-updates](https://apstudents.collegeboard.org/coronavirus-updates) |
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1.1. Variable Expressions

Learning Objectives

- Evaluate algebraic expressions.
- Evaluate algebraic expressions with exponents.

Introduction - The Language of Algebra

No one likes doing the same problem over and over again—which’s why mathematicians invented algebra. Algebra takes the basic principles of math and makes them more general, so we can solve a problem once and then use that solution to solve a group of similar problems.

In arithmetic, you’ve dealt with numbers and their arithmetical operations (such as $+$, $-$, $\times$, $\div$). In algebra, we use symbols called variables (which are usually letters, such as $x$, $y$, $a$, $b$, $c$, ...) to represent numbers and sometimes processes.

For example, we might use the letter $x$ to represent some number we don’t know yet, which we might need to figure out in the course of a problem. Or we might use two letters, like $x$ and $y$, to show a relationship between two numbers without needing to know what the actual numbers are. The same letters can represent a wide range of possible numbers, and the same letter may represent completely different numbers when used in two different problems.

Using variables offers advantages over solving each problem “from scratch.” With variables, we can:

- Formulate arithmetical laws such as $a + b = b + a$ for all real numbers $a$ and $b$.
- Refer to “unknown” numbers. For instance: find a number $x$ such that $3x + 1 = 10$.
- Write more compactly about functional relationships such as, “If you sell $x$ tickets, then your profit will be $3x - 10$ dollars, or $f(x) = 3x - 10$,” where “$f$” is the profit function, and $x$ is the input (i.e. how many tickets you sell).

Example 1

Write an algebraic expression for the perimeter and area of the rectangle below.

To find the perimeter, we add the lengths of all 4 sides. We can still do this even if we don’t know the side lengths in numbers, because we can use variables like $l$ and $w$ to represent the unknown length and width. If we start at the top left and work clockwise, and if we use the letter $P$ to represent the perimeter, then we can say:
$P = l + w + l + w$

We are adding 2 $l$’s and 2 $w$’s, so we can say that:

$P = 2 \cdot l + 2 \cdot w$

It’s customary in algebra to omit multiplication symbols whenever possible. For example, $11x$ means the same thing as $11 \cdot x$ or $11 \times x$. We can therefore also write:

$P = 2l + 2w$

Area is length multiplied by width. In algebraic terms we get:

$A = l \times w \rightarrow A = l \cdot w \rightarrow A = lw$

Note: $2l + 2w$ by itself is an example of a variable expression; $P = 2l + 2w$ is an example of an equation. The main difference between expressions and equations is the presence of an equals sign (=).

In the above example, we found the simplest possible ways to express the perimeter and area of a rectangle when we don’t yet know what its length and width actually are. Now, when we encounter a rectangle whose dimensions we do know, we can simply substitute (or plug in) those values in the above equations. In this chapter, we will encounter many expressions that we can evaluate by plugging in values for the variables involved.

### Evaluate Algebraic Expressions

When we are given an algebraic expression, one of the most common things we might have to do with it is evaluate it for some given value of the variable. The following example illustrates this process.

**Example 2**

Let $x = 12$. Find the value of $2x - 7$.

To find the solution, we substitute 12 for $x$ in the given expression. Every time we see $x$, we replace it with 12.

\[
2x - 7 = 2(12) - 7 \\
= 24 - 7 \\
= 17
\]

Note: At this stage of the problem, we place the substituted value in parentheses. We do this to make the written-out problem easier to follow, and to avoid mistakes. (If we didn’t use parentheses and also forgot to add a multiplication sign, we would end up turning $2x$ into 212 instead of 2 times 12!)

**Example 3**

Let $y = -2$. Find the value of $\frac{7}{y} - 11y + 2$.
Solution

\[
\frac{7}{(-2)} - 11(-2) + 2 = -3\frac{1}{2} + 22 + 2 \\
= 24 - 3\frac{1}{2} \\
= 20\frac{1}{2}
\]

Many expressions have more than one variable in them. For example, the formula for the perimeter of a rectangle in the introduction has two variables: length \((l)\) and width \((w)\). In these cases, be careful to substitute the appropriate value in the appropriate place.

**Example 4**

![Diagram of a trapezoid with labels](image)

The area of a trapezoid is given by the equation \(A = \frac{h}{2}(a + b)\). Find the area of a trapezoid with bases \(a = 10\) cm and \(b = 15\) cm and height \(h = 8\) cm.

To find the solution to this problem, we simply take the values given for the variables \(a\), \(b\), and \(h\), and plug them in to the expression for \(A\):

\[
A = \frac{h}{2}(a + b) \quad \text{Substitute 10 for } a, 15 \text{ for } b, \text{ and } 8 \text{ for } h.
\]

\[
A = \frac{8}{2}(10 + 15) \quad \text{Evaluate piece by piece. } 10 + 15 = 25; \quad \frac{8}{2} = 4.
\]

\[
A = 4(25) = 100
\]

**Solution:** The area of the trapezoid is 100 square centimeters.

---

### Evaluate Algebraic Expressions with Exponents

Many formulas and equations in mathematics contain exponents. Exponents are used as a short-hand notation for repeated multiplication. For example:

\[
2 \cdot 2 = 2^2
\]

\[
2 \cdot 2 \cdot 2 = 2^3
\]
The exponent stands for how many times the number is used as a factor (multiplied). When we deal with integers, it is usually easiest to simplify the expression. We simplify:

\[ 2^2 = 4 \]
\[ 2^3 = 8 \]

However, we need exponents when we work with variables, because it is much easier to write \( x^8 \) than \( x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \).

To evaluate expressions with exponents, substitute the values you are given for each variable and simplify. It is especially important in this case to substitute using parentheses in order to make sure that the simplification is done correctly.


**Example 5**

The area of a circle is given by the formula \( A = \pi r^2 \). Find the area of a circle with radius \( r = 17 \) inches.

Substitute values into the equation.

\[
A = \pi r^2 \quad \text{Substitute } 17 \text{ for } r.
A = \pi (17)^2 \quad \pi \cdot 17 \cdot 17 \approx 907.9202\ldots \text{ Round to 2 decimal places.}
\]

The area is approximately 907.92 square inches.

**Example 6**

Find the value of \( \frac{x^2y^3}{x^3+y^2} \), for \( x = 2 \) and \( y = -4 \).

Substitute the values of \( x \) and \( y \) in the following.

\[
\frac{x^2y^3}{x^3+y^2} = \frac{(2)^2(-4)^3}{(2)^3 + (-4)^2} \]
\[
\frac{4(-64)}{8 + 16} = -\frac{256}{24} = -\frac{32}{3}
\]

Substitute 2 for \( x \) and \(-4\) for \( y \).

Evaluate expressions: \( (2)^2 = (2)(2) = 4 \) and \( (2)^3 = (2)(2)(2) = 8 \), \( (-4)^2 = (-4)(-4) = 16 \) and \( (-4)^3 = (-4)(-4)(-4) = -64 \).

**Example 7**
The height \((h)\) of a ball in flight is given by the formula \(h = -32t^2 + 60t + 20\), where the height is given in feet and the time \((t)\) is given in seconds. Find the height of the ball at time \(t = 2\) seconds.

**Solution**

\[
h = -32t^2 + 60t + 20
\]
\[
= -32(2)^2 + 60(2) + 20 \quad \text{Substitute 2 for } t.
\]
\[
= -32(4) + 60(2) + 20
\]
\[
= 12
\]

The height of the ball is 12 feet.

**Review Questions**

1. Write the following in a more condensed form by leaving out a multiplication symbol.
   a. \(2 \times 11x\)
   b. \(1.35 \cdot y\)
   c. \(3 \times \frac{1}{4}\)
   d. \(\frac{1}{2} \cdot z\)

2. Evaluate the following expressions for \(a = -3\), \(b = 2\), \(c = 5\), and \(d = -4\).
   a. \(2a + 3b\)
   b. \(4c + d\)
   c. \(5ac - 2b\)
   d. \(\frac{2a}{c-d}\)
   e. \(\frac{3b}{d}\)
   f. \(\frac{a-4b}{5c+2d}\)
   g. \(\frac{1}{a+b}\)
   h. \(\frac{ab}{cd}\)

3. Evaluate the following expressions for \(x = -1\), \(y = 2\), \(z = -3\), and \(w = 4\).
   a. \(8x^3\)
   b. \(\frac{5z^3}{6w^3}\)
   c. \(3z^2 - 5w^2\)
   d. \(x^2 - y^2\)
   e. \(\frac{z^3 + w^3}{z^2 - w^3}\)
   f. \(2x^3 - 3x^2 + 5x - 4\)
   g. \(4w^3 + 3w^2 - w + 2\)
   h. \(3 + \frac{1}{z^2}\)

4. The weekly cost \(C\) of manufacturing \(x\) remote controls is given by the formula \(C = 2000 + 3x\), where the cost is given in dollars.
   a. What is the cost of producing 1000 remote controls?
   b. What is the cost of producing 2000 remote controls?
   c. What is the cost of producing 2500 remote controls?

5. The volume of a box without a lid is given by the formula \(V = 4x(10 - x)^2\), where \(x\) is a length in inches and \(V\) is the volume in cubic inches.
a. What is the volume when $x = 2$?
b. What is the volume when $x = 3$?
1.2 Order of Operations

Learning Objectives

- Evaluate algebraic expressions with grouping symbols.
- Evaluate algebraic expressions with fraction bars.
- Evaluate algebraic expressions using a graphing calculator.

Introduction

Look at and evaluate the following expression:

\[ 2 + 4 \times 7 - 1 \]

How many different ways can we interpret this problem, and how many different answers could someone possibly find for it?

The simplest way to evaluate the expression is simply to start at the left and work your way across:

\[
\begin{align*}
2 + 4 \times 7 - 1 & = ? \\
& = 6 \times 7 - 1 \\
& = 42 - 1 \\
& = 41
\end{align*}
\]

This is the answer you would get if you entered the expression into an ordinary calculator. But if you entered the expression into a scientific calculator or a graphing calculator you would probably get 29 as the answer.

In mathematics, the order in which we perform the various operations (such as adding, multiplying, etc.) is important. In the expression above, the operation of multiplication takes precedence over addition, so we evaluate it first. Let’s re-write the expression, but put the multiplication in brackets to show that it is to be evaluated first.

\[ 2 + (4 \times 7) - 1 = ? \]

First evaluate the brackets: \(4 \times 7 = 28\). Our expression becomes:

\[ 2 + (28) - 1 = ? \]

When we have only addition and subtraction, we start at the left and work across:
2 + 28 - 1
= 30 - 1
= 29

Algebra students often use the word “PEMDAS” to help remember the order in which we evaluate the mathematical expressions: Parentheses, Exponents, Multiplication, Division, Addition and Subtraction.

Order of Operations

1. Evaluate expressions within Parentheses (also all brackets [ ] and braces { }) first.
2. Evaluate all Exponents (terms such as $3^2$ or $x^3$) next.
3. Multiplication and Division is next - work from left to right completing both multiplication and division in the order that they appear.
4. Finally, evaluate Addition and Subtraction - work from left to right completing both addition and subtraction in the order that they appear.

Evaluate Algebraic Expressions with Grouping Symbols

The first step in the order of operations is called parentheses, but we include all grouping symbols in this step—not just parentheses ( ), but also square brackets [ ] and curly braces { }.

Example 1
Evaluate the following:

a) $4 - 7 - 11 + 2$

b) $4 - (7 - 11) + 2$

c) $4 - [7 - (11 + 2)]$

Each of these expressions has the same numbers and the same mathematical operations, in the same order. The placement of the various grouping symbols means, however, that we must evaluate everything in a different order each time. Let’s look at how we evaluate each of these examples.

a) This expression doesn’t have parentheses, exponents, multiplication, or division. PEMDAS states that we treat addition and subtraction as they appear, starting at the left and working right (it’s NOT addition then subtraction).

$$4 - 7 - 11 + 2 = -3 - 11 + 2$$
$$= -14 + 2$$
$$= -12$$

b) This expression has parentheses, so we first evaluate $7 - 11 = -4$. Remember that when we subtract a negative it is equivalent to adding a positive:
1.2. Order of Operations

4 − (7 − 11) + 2 = 4 − (−4) + 2
= 8 + 2
= 10

c) An expression can contain any number of sets of parentheses. Sometimes expressions will have sets of parentheses inside other sets of parentheses. When faced with nested parentheses, start at the innermost parentheses and work outward.

Brackets may also be used to group expressions which already contain parentheses. This expression has both brackets and parentheses. We start with the innermost group: 11 + 2 = 13. Then we complete the operation in the brackets.

4 − [7 − (11 + 2)] = 4 − [7 − (13)]
= 4 − [−6]
= 10

Example 2

Evaluate the following:

a) 3 × 5 − 7 ÷ 2

b) 3 × (5 − 7) ÷ 2

c) (3 × 5) − (7 ÷ 2)

a) There are no grouping symbols. PEMDAS dictates that we multiply and divide first, working from left to right: 3 × 5 = 15 and 7 ÷ 2 = 3.5. (NOTE: It’s not multiplication then division.) Next we subtract:

3 × 5 − 7 ÷ 2 = 15 − 3.5
= 11.5

b) First, we evaluate the expression inside the parentheses: 5 − 7 = −2. Then work from left to right:

3 × (5 − 7) ÷ 2 = 3 × (−2) ÷ 2
= (−6) ÷ 2
= −3

c) First, we evaluate the expressions inside parentheses: 3 × 5 = 15 and 7 ÷ 2 = 3.5. Then work from left to right:

(3 × 5) − (7 ÷ 2) = 15 − 3.5
= 11.5

Note that adding parentheses didn’t change the expression in part c, but did make it easier to read. Parentheses can be used to change the order of operations in an expression, but they can also be used simply to make it easier to understand.
We can also use the order of operations to simplify an expression that has variables in it, after we substitute specific values for those variables.

**Example 3**

*Use the order of operations to evaluate the following:*

a) $2 - (3x + 2)$ when $x = 2$

b) $3y^2 + 2y + 1$ when $y = -3$

c) $2 - (t - 7)^2 \times (u^3 - v)$ when $t = 19$, $u = 4$, and $v = 2$

**a)** The first step is to substitute the value for $x$ into the expression. We can put it in parentheses to clarify the resulting expression.

$$2 - (3(2) + 2)$$

(Note: $3(2)$ is the same as $3 \times 2$.)

Follow **PEMDAS** - first parentheses. Inside parentheses follow **PEMDAS** again.

$$2 - (3 \times 2 + 2) = 2 - (6 + 2) \quad \text{Inside the parentheses, we multiply first.}$$

$$2 - 8 = -6 \quad \text{Next we add inside the parentheses, and finally we subtract.}$$

**b)** The first step is to substitute the value for $y$ into the expression.

$$3 \times (-3)^2 + 2 \times (-3) - 1$$

Follow **PEMDAS**: we cannot simplify the expressions in parentheses, so exponents come next.

$$3 \times (-3)^2 + 2 \times (-3) - 1 \quad \text{Evaluate exponents: } (-3)^2 = 9$$
$$= 3 \times 9 + 2 \times (-3) - 1 \quad \text{Evaluate multiplication: } 3 \times 9 = 27; \ 2 \times -3 = -6$$
$$= 27 + (-6) - 1 \quad \text{Add and subtract in order from left to right.}$$
$$= 27 - 6 - 1$$
$$= 20$$

c) The first step is to substitute the values for $t$, $u$, and $v$ into the expression.

$$2 - (19 - 7)^2 \times (4^3 - 2)$$

Follow **PEMDAS**:

$$2 - (19 - 7)^2 \times (4^3 - 2) \quad \text{Evaluate parentheses: } (19 - 7) = 12; \ (4^3 - 2) = (64 - 2) = 62$$
$$= 2 - 12^2 \times 62 \quad \text{Evaluate exponents: } 12^2 = 144$$
$$= 2 - 144 \times 62 \quad \text{Multiply: } 144 \times 62 = 8928$$
$$= 2 - 8928 \quad \text{Subtract.}$$
$$= -8926$$
In parts (b) and (c) we left the parentheses around the negative numbers to clarify the problem. They did not affect the order of operations, but they did help avoid confusion when we were multiplying negative numbers.

Part (c) in the last example shows another interesting point. When we have an expression inside the parentheses, we use **PEMDAS** to determine the order in which we evaluate the contents.

---

### Evaluate Algebraic Expressions with Fraction Bars

Fraction bars count as grouping symbols for **PEMDAS**, so we evaluate them in the first step of solving an expression. All numerators and all denominators can be treated as if they have invisible parentheses around them. When **real** parentheses are also present, remember that the innermost grouping symbols come first. If, for example, parentheses appear on a numerator, they would take precedence over the fraction bar. If the parentheses appear outside of the fraction, then the fraction bar takes precedence.

**Example 4**

*Use the order of operations to evaluate the following expressions:*

a) \( \frac{z+3}{4} - 1 \) when \( z = 2 \)

b) \( \left( \frac{a+2}{b+4} - 1 \right) + b \) when \( a = 3 \) and \( b = 1 \)

c) \( 2 \times \left( \frac{w+x-2z}{y+2} \right) - 1 \) when \( w = 11, x = 3, y = 1, \) and \( z = -2 \)

**a)** We substitute the value for \( z \) into the expression.

\[
\frac{2 + 3}{4} - 1
\]

Although this expression has no parentheses, the fraction bar is also a grouping symbol—it has the same effect as a set of parentheses. We can write in the “invisible parentheses” for clarity:

\[
\frac{(2 + 3)}{4} - 1
\]

Using **PEMDAS**, we first evaluate the numerator:

\[
\frac{5}{4} - 1
\]

We can convert \( \frac{5}{4} \) to a mixed number:

\[
\frac{5}{4} = 1 \frac{1}{4}
\]

Then evaluate the expression:

\[
1 \frac{1}{4} - 1 = \frac{1}{4}
\]
b) We substitute the values for \(a\) and \(b\) into the expression:

\[
\left(\frac{3 + 2}{1 + 4} - 1\right) + 1
\]

This expression has nested parentheses (remember the effect of the fraction bar). The innermost grouping symbol is provided by the fraction bar. We evaluate the numerator \((3 + 2)\) and denominator \((1 + 4)\) first.

\[
\left(\frac{3 + 2}{1 + 4} - 1\right) + 1 = \left(\frac{5 - 1}{1 - 1}\right) + 1
\]

Next we evaluate the inside of the parentheses. First we divide.

\[
\frac{5}{1} - 1 + 1
\]

Next we subtract.

\[
0 + 1 = 1
\]

c) We substitute the values for \(w, x, y,\) and \(z\) into the expression:

\[
2 \times \left(\frac{11 + (3 - 2(-2))}{(1 + 2)^2} - 1\right)
\]

This complicated expression has several layers of nested parentheses. One method for ensuring that we start with the innermost parentheses is to use more than one type of parentheses. Working from the outside, we can leave the outermost brackets as parentheses \(()\). Next will be the “invisible brackets” from the fraction bar; we will write these as \(\left[\right]\). The third level of nested parentheses will be the \(\{\}\). We will leave negative numbers in round brackets.

\[
2 \times \left(\frac{\left[11 + (3 - 2(-2))\right]}{\{1 + 2\}^2} - 1\right)
\]

Start with the innermost grouping sign: \(\{\}\).

\[
\{1 + 2\} = 3; \quad \{3 - 2(-2)\} = 3 + 4 = 7
\]

Next, evaluate the square brackets.

\[
\frac{11 + 7}{3^2} - 1
\]

Next, evaluate the round brackets. Start with division.

\[
\frac{18}{9} - 1
\]

Finally, do the addition and subtraction.

\[
2(2 - 1) = 2
\]

Evaluate Algebraic Expressions with a TI-83/84 Family Graphing Calculator

A graphing calculator is a very useful tool in evaluating algebraic expressions. Like a scientific calculator, a graphing calculator follows **PEMDAS**. In this section we will explain two ways of evaluating expressions with the graphing calculator.

**Example 5**

**Evaluate** \(3(x^2 - 1)^2 - x^4 + 12\) \(+ 5x^3 - 1\) **when** \(x = -3\).

**Method 1:** Substitute for the variable first. Then evaluate the numerical expression with the calculator.

**Substitute the value** \(x = -3\) **into the expression.**

\[
3((-3)^2 - 1)^2 - (-3)^4 + 12 + 5(-3)^3 - 1
\]
1.2. Order of Operations

Input this in the calculator just as it is and press [ENTER]. (Note: use ∧ to enter exponents)

\[ \frac{3(3^2-1)^2-(-3)^4+1}{2+5(-3)^3-1} \]

The answer is -13.

**Method 2:** Input the original expression in the calculator first and then evaluate.

First, store the value \( x = -3 \) in the calculator. Type -3 [STO] x (The letter x can be entered using the x−[VAR] button or [ALPHA] + [STO]). Then type the original expression in the calculator and press [ENTER].

The answer is -13.

The second method is better because you can easily evaluate the same expression for any value you want. For example, let’s evaluate the same expression using the values \( x = 2 \) and \( x = \frac{2}{3} \).

For \( x = 2 \), store the value of \( x \) in the calculator: 2 [STO] x. Press [2nd] [ENTER] twice to get the previous expression you typed in on the screen without having to enter it again. Press [ENTER] to evaluate the expression.

The answer is 62.

For \( x = \frac{2}{3} \), store the value of \( x \) in the calculator: \( \frac{2}{3} \) [STO] x. Press [2nd] [ENTER] twice to get the expression on the screen without having to enter it again. Press [ENTER] to evaluate.

The answer is 13.21, or \( \frac{1070}{81} \) in fraction form.

**Note:** On graphing calculators there is a difference between the minus sign and the negative sign. When we stored the value negative three, we needed to use the negative sign which is to the left of the [ENTER] button on the
calculator. On the other hand, to perform the subtraction operation in the expression we used the minus sign. The minus sign is right above the plus sign on the right.

You can also use a graphing calculator to evaluate expressions with more than one variable.

**Example 7**

*Evaluate the expression* \( \frac{3x^2-4y^2+x^4}{(x+y)^2} \) *for* \( x = 2, y = -1 \).

**Solution**

Store the values of \( x \) and \( y \): 2 [STO] \( x \), -1 [STO] \( y \). (The letters \( x \) and \( y \) can be entered using [ALPHA] + [KEY].) Input the expression in the calculator. When an expression includes a fraction, be sure to use parentheses: (numerator) / (denominator).

Press [ENTER] to obtain the answer 24.

**Additional Resources**

For more practice, you can play an algebra game involving order of operations online at [http://www.funbrain.com/algebra/index.html](http://www.funbrain.com/algebra/index.html).

**Review Questions**

1. Use the order of operations to evaluate the following expressions.
   a. \( 8 - (19 - (2 + 5) - 7) \)
   b. \( 2 + 7 \times 11 - 12 \div 3 \)
   c. \( (3 + 7) \div (7 - 12) \)
   d. \( \frac{2(3+2-1)}{4-6+1} - (3 - 5) \)
   e. \( \frac{4+7(3)}{9-4} + \frac{12-3.2}{2} \)
   f. \( (4 - 1)^2 + 3^2 \cdot 2 \)
   g. \( \frac{(2^2+5)^2}{3^2-4^2} \div (2 + 1) \)

2. Evaluate the following expressions involving variables.
   a. \( \frac{jk}{j+k} \) when \( j = 6 \) and \( k = 12 \)
   b. \( 2y^2 \) when \( x = 1 \) and \( y = 5 \)
   c. \( 3x^2 + 2x + 1 \) when \( x = 5 \)
   d. \( (y^2 - x)^2 \) when \( x = 2 \) and \( y = 1 \)
   e. \( \frac{x+y^2}{3-x} \) when \( x = 2 \) and \( y = 3 \)
3. Evaluate the following expressions involving variables.
   a. $\frac{4x}{2x^2-3x+1}$ when $x = 2$
   b. $\frac{x^2}{x+y} + \frac{x}{x-y}$ when $x = 1$, $y = -2$, and $z = 4$
   c. $\frac{4xyz}{y^2-x}$ when $x = 3$, $y = 2$, and $z = 5$
   d. $\frac{x^2-x^2}{xz-2(x^2-x)}$ when $x = -1$ and $z = 3$

4. Insert parentheses in each expression to make a true equation.
   a. $5 - 2 \times 6 - 5 + 2 = 5$
   b. $12 \div 4 + 10 - 3 \times 3 + 7 = 11$
   c. $22 - 32 - 5 \times 3 - 5 = 30$
   d. $12 - 8 - 4 \times 5 = -8$

5. Evaluate each expression using a graphing calculator.
   a. $x^2 + 2x - xy$ when $x = 250$ and $y = -120$
   b. $(xy - y^4)^2$ when $x = 0.02$ and $y = -0.025$
   c. $\frac{x+y-z}{xy+z}$ when $x = \frac{1}{2}$, $y = \frac{3}{2}$, and $z = -1$
   d. $\frac{(x+y)^2}{4x^2-xy}$ when $x = 3$ and $y = -5$
   e. $\frac{(x-y)^3}{x^3-y} + \frac{(x+y)^2}{x^2+y}$ when $x = 4$ and $y = -2$
1.3 Patterns and Equations

Learning Objectives

- Write an equation.
- Use a verbal model to write an equation.
- Solve problems using equations.

Introduction

In mathematics, and especially in algebra, we look for patterns in the numbers we see. The tools of algebra help us describe these patterns with words and with equations (formulas or functions). An equation is a mathematical recipe that gives the value of one variable in terms of another.

For example, if a theme park charges $12 admission, then the number of people who enter the park every day and the amount of money taken in by the ticket office are related mathematically, and we can write a rule to find the amount of money taken in by the ticket office.

In words, we might say “The amount of money taken in is equal to twelve times the number of people who enter the park.”

We could also make a table. The following table relates the number of people who visit the park and the total money taken in by the ticket office.

<table>
<thead>
<tr>
<th>Number of visitors</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money taken in ($)</td>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
<td>72</td>
<td>84</td>
</tr>
</tbody>
</table>

Clearly, we would need a big table to cope with a busy day in the middle of a school vacation!

A third way we might relate the two quantities (visitors and money) is with a graph. If we plot the money taken in on the vertical axis and the number of visitors on the horizontal axis, then we would have a graph that looks like the one shown below. Note that this graph shows a smooth line that includes non-whole number values of $x$ (e.g. $x = 2.5$). In real life this would not make sense, because fractions of people can’t visit a park. This is an issue of domain and range, something we will talk about later.
The method we will examine in detail in this lesson is closer to the first way we chose to describe the relationship. In words we said that “The amount of money taken in is twelve times the number of people who enter the park.” In mathematical terms we can describe this sort of relationship with variables. A variable is a letter used to represent an unknown quantity. We can see the beginning of a mathematical formula in the words:

The amount of money taken in is twelve times the number of people who enter the park.

This can be translated to:

\[
\text{the amount of money taken in} = 12 \times (\text{the number of people who enter the park})
\]

We can now see which quantities can be assigned to letters. First we must state which letters (or variables) relate to which quantities. We call this defining the variables:

Let \( x \) = the number of people who enter the theme park.

Let \( y \) = the total amount of money taken in at the ticket office.

We now have a fourth way to describe the relationship: with an algebraic equation.

\[
y = 12x
\]

Writing a mathematical equation using variables is very convenient. You can perform all of the operations necessary to solve this problem without having to write out the known and unknown quantities over and over again. At the end of the problem, you just need to remember which quantities \( x \) and \( y \) represent.

---

Write an Equation

An equation is a term used to describe a collection of numbers and variables related through mathematical operators. An algebraic equation will contain letters that represent real quantities. For example, if we wanted to use the algebraic equation in the example above to find the money taken in for a certain number of visitors, we would substitute that number for \( x \) and then solve the resulting equation for \( y \).

Example 1
A theme park charges $12 entry to visitors. Find the money taken in if 1296 people visit the park.

Let’s break the solution to this problem down into steps. This will be a useful strategy for all the problems in this lesson.

**Step 1:** Extract the important information.

\[
\text{(number of dollars taken in)} = 12 \times \text{(number of visitors)}
\]

\[
\text{(number of visitors)} = 1296
\]

**Step 2:** Translate into a mathematical equation. To do this, we pick variables to stand for the numbers.

Let \( y = \text{(number of dollars taken in)} \).

Let \( x = \text{(number of visitors)} \).

\[
\text{(number of dollars taken in)} = 12 \times \text{(number of visitors)}
\]

\[
y = 12 \times x
\]

**Step 3:** Substitute in any known values for the variables.

\[
y = 12 \times x
\]

\[
x = 1296
\]

\[
\therefore
\]

\[
y = 12 \times 1296
\]

**Step 4:** Solve the equation.

\[
y = 12 \times 1296 = 15552
\]

The amount of money taken in is $15552.

**Step 5:** Check the result.

If $15552 is taken at the ticket office and tickets are $12, then we can divide the total amount of money collected by the price per individual ticket.

\[
\text{(number of people)} = \frac{15552}{12} = 1296
\]

1296 is indeed the number of people who entered the park. **The answer checks out.**

**Example 2**

The following table shows the relationship between two quantities. First, write an equation that describes the relationship. Then, find out the value of \( b \) when \( a \) is 750.
Step 1: Extract the important information.

We can see from the table that every time \( a \) increases by 10, \( b \) increases by 20. However, \( b \) is not simply twice the value of \( a \). We can see that when \( a = 0 \), \( b = 20 \), and this gives a clue as to what rule the pattern follows. The rule linking \( a \) and \( b \) is:

“To find \( b \), double the value of \( a \) and add 20.”

Step 2: Translate into a mathematical equation:

<table>
<thead>
<tr>
<th>Text</th>
<th>Translates to</th>
<th>Mathematical Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>“To find ( b )”</td>
<td>( \rightarrow )</td>
<td>( b = )</td>
</tr>
<tr>
<td>“double the value of ( a )”</td>
<td>( \rightarrow )</td>
<td>( 2a )</td>
</tr>
<tr>
<td>“add 20”</td>
<td>( \rightarrow )</td>
<td>( + 20 )</td>
</tr>
</tbody>
</table>

Our equation is \( b = 2a + 20 \).

Step 3: Solve the equation.

The original problem asks for the value of \( b \) when \( a \) is 750. When \( a \) is 750, \( b = 2a + 20 \) becomes \( b = 2(750) + 20 \). Following the order of operations, we get:

\[
b = 2(750) + 20
= 1500 + 20
= 1520
\]

Step 4: Check the result.

In some cases you can check the result by plugging it back into the original equation. Other times you must simply double-check your math. In either case, checking your answer is always a good idea. In this case, we can plug our answer for \( b \) into the equation, along with the value for \( a \), and see what comes out. \( 1520 = 2(750) + 20 \) is TRUE because both sides of the equation are equal. A true statement means that the answer checks out.

Use a Verbal Model to Write an Equation

In the last example we developed a rule, written in words, as a way to develop an algebraic equation. We will develop this further in the next few examples.

Example 3

The following table shows the values of two related quantities. Write an equation that describes the relationship mathematically.
**Step 1: Extract the important information.**

We can see from the table that \(y\) is five times bigger than \(x\). The value for \(y\) is negative when \(x\) is positive, and it is positive when \(x\) is negative. Here is the rule that links \(x\) and \(y\):

“\(y\) is the negative of five times the value of \(x\)”

**Step 2: Translate this statement into a mathematical equation.**

**Table 1.3:**

<table>
<thead>
<tr>
<th>Text</th>
<th>Translates to</th>
<th>Mathematical Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>“(y) is”</td>
<td>(\rightarrow)</td>
<td>(y = )</td>
</tr>
<tr>
<td>“negative 5 times the value of (x)”</td>
<td>(\rightarrow)</td>
<td>(-5x)</td>
</tr>
</tbody>
</table>

Our equation is \(y = -5x\).

**Step 3: There is nothing in this problem to solve for. We can move to Step 4.**

**Step 4: Check the result.**

In this case, the way we would check our answer is to use the equation to generate our own \(xy\) pairs. If they match the values in the table, then we know our equation is correct. We will plug in -2, 0, 2, 4, and 6 for \(x\) and solve for \(y\):

**Table 1.4:**

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>(-5(-2) = 10)</td>
</tr>
<tr>
<td>0</td>
<td>(-5(0) = 0)</td>
</tr>
<tr>
<td>2</td>
<td>(-5(2) = -10)</td>
</tr>
<tr>
<td>4</td>
<td>(-5(4) = -20)</td>
</tr>
<tr>
<td>6</td>
<td>(-5(6) = -30)</td>
</tr>
</tbody>
</table>

The \(y\)–values in this table match the ones in the earlier table. **The answer checks out.**

**Example 4**

*Zarina has a $100 gift card, and she has been spending money on the card in small regular amounts. She checks the balance on the card weekly and records it in the following table.*

**Table 1.5:**

<table>
<thead>
<tr>
<th>Week Number</th>
<th>Balance ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>78</td>
</tr>
<tr>
<td>3</td>
<td>56</td>
</tr>
<tr>
<td>4</td>
<td>34</td>
</tr>
</tbody>
</table>
Write an equation for the money remaining on the card in any given week.

Step 1: Extract the important information.

The balance remaining on the card is not just a constant multiple of the week number; 100 is 100 times 1, but 78 is not 100 times 2. But there is still a pattern: the balance decreases by 22 whenever the week number increases by 1. This suggests that the balance is somehow related to the amount “-22 times the week number.”

In fact, the balance equals “-22 times the week number, plus something.” To determine what that something is, we can look at the values in one row on the table—for example, the first row, where we have a balance of $100 for week number 1.

Step 2: Translate into a mathematical equation.

First, we define our variables. Let \( n \) stand for the week number and \( b \) for the balance.

Then we can translate our verbal expression as follows:

<table>
<thead>
<tr>
<th>Text</th>
<th>Translates to</th>
<th>Mathematical Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance equals -22 times the week number, plus something.</td>
<td>( b = -22n + ? )</td>
<td></td>
</tr>
</tbody>
</table>

To find out what that \( ? \) represents, we can plug in the values from that first row of the table, where \( b = 100 \) and \( n = 1 \). This gives us \( 100 = -22(1) + ? \).

So what number gives 100 when you add -22 to it? The answer is 122, so that is the number the \( ? \) stands for. Now our final equation is:

\[
b = -22n + 122
\]

Step 3: All we were asked to find was the expression. We weren’t asked to solve it, so we can move to Step 4.

Step 4: Check the result.

To check that this equation is correct, we see if it really reproduces the data in the table. To do that we plug in values for \( n \):

\[
\begin{align*}
n = 1 & \rightarrow b = -22(1) + 122 = 122 - 22 = 100 \\
n = 2 & \rightarrow b = -22(2) + 122 = 122 - 44 = 78 \\
n = 3 & \rightarrow b = -22(3) + 122 = 122 - 66 = 56 \\
n = 4 & \rightarrow b = -22(4) + 122 = 122 - 88 = 34
\end{align*}
\]

The equation perfectly reproduces the data in the table. The answer checks out.

Solve Problems Using Equations

Let’s solve the following real-world problem by using the given information to write a mathematical equation that can be solved for a solution.

Example 5
A group of students are in a room. After 25 students leave, it is found that \( \frac{2}{3} \) of the original group is left in the room. How many students were in the room at the start?

**Step 1:** Extract the important information

We know that 25 students leave the room.

We know that \( \frac{2}{3} \) of the original number of students are left in the room.

We need to find how many students were in the room at the start.

**Step 2:** Translate into a mathematical equation. Initially we have an unknown number of students in the room. We can refer to this as the original number.

Let’s define the variable \( x = \) the original number of students in the room. After 25 students leave the room, the number of students in the room is \( x - 25 \). We also know that the number of students left is \( \frac{2}{3} \) of \( x \). So we have two expressions for the number of students left, and those two expressions are equal because they represent the same number. That means our equation is:

\[
\frac{2}{3}x = x - 25
\]

**Step 3:** Solve the equation.

*Add 25 to both sides.*

\[
x - 25 = \frac{2}{3}x
\]

\[
x - 25 + 25 = \frac{2}{3}x + 25
\]

\[
x = \frac{2}{3}x + 25
\]

*Subtract \( \frac{2}{3}x \) from both sides.*

\[
x - \frac{2}{3}x = \frac{2}{3}x - \frac{2}{3}x + 25
\]

\[
\frac{1}{3}x = 25
\]

*Multiply both sides by 3.*

\[
3 \cdot \frac{1}{3}x = 3 \cdot 25
\]

\[
x = 75
\]

Remember that \( x \) represents the original number of students in the room. So, there were 75 students in the room to start with.

**Step 4:** Check the answer:

If we start with 75 students in the room and 25 of them leave, then there are \( 75 - 25 = 50 \) students left in the room. \( \frac{2}{5} \) of the original number is \( \frac{2}{5} \cdot 75 = 50 \).
This means that the number of students who are left over equals \( \frac{2}{3} \) of the original number. The answer checks out. The method of defining variables and writing a mathematical equation is the method you will use the most in an algebra course. This method is often used together with other techniques such as making a table of values, creating a graph, drawing a diagram and looking for a pattern.

### Review Questions

**Table 1.7:**

<table>
<thead>
<tr>
<th>Day</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
</tr>
</tbody>
</table>

1. The above table depicts the profit in dollars taken in by a store each day.
   a. Write a mathematical equation that describes the relationship between the variables in the table.
   b. What is the profit on day 10?
   c. If the profit on a certain day is $200, what is the profit on the next day?

2. a. Write a mathematical equation that describes the situation: *A full cookie jar has 24 cookies. How many cookies are left in the jar after you have eaten some?*
   b. How many cookies are in the jar after you have eaten 9 cookies?
   c. How many cookies are in the jar after you have eaten 9 cookies and then eaten 3 more?

3. Write a mathematical equation for the following situations and solve.
   a. Seven times a number is 35. What is the number?
   b. Three times a number, plus 15, is 24. What is the number?
   c. Twice a number is three less than five times another number. Three times the second number is 15. What are the numbers?
   d. One number is 25 more than 2 times another number. If each number were multiplied by five, their sum would be 350. What are the numbers?
   e. The sum of two consecutive integers is 35. What are the numbers?
   f. Peter is three times as old as he was six years ago. How old is Peter?

4. How much water should be added to one liter of pure alcohol to make a mixture of 25% alcohol?

5. A mixture of 50% alcohol and 50% water has 4 liters of water added to it. It is now 25% alcohol. What was the total volume of the original mixture?

6. In Crystal’s silverware drawer there are twice as many spoons as forks. If Crystal adds nine forks to the drawer, there will be twice as many forks as spoons. How many forks and how many spoons are in the drawer right now?

7. a. Mia drove to Javier’s house at 40 miles per hour. Javier’s house is 20 miles away. Mia arrived at Javier’s house at 2:00 pm. What time did she leave?
   b. Mia left Javier’s house at 6:00 pm to drive home. This time she drove 25% faster. What time did she arrive home?
   c. The next day, Mia took the expressway to Javier’s house. This route was 24 miles long, but she was able to drive at 60 miles per hour. How long did the trip take?
d. When Mia took the same route back, traffic on the expressway was 20% slower. How long did the return trip take?

8. The price of an mp3 player decreased by 20% from last year to this year. This year the price of the player is $120. What was the price last year?

9. SmartCo sells deluxe widgets for $60 each, which includes the cost of manufacture plus a 20% markup. What does it cost SmartCo to manufacture each widget?

10. Jae just took a math test with 20 questions, each worth an equal number of points. The test is worth 100 points total.

   a. Write an equation relating the number of questions Jae got right to the total score he will get on the test.
   b. If a score of 70 points earns a grade of C-, how many questions would Jae need to get right to get a C- on the test?
   c. If a score of 83 points earns a grade of B, how many questions would Jae need to get right to get a B on the test?
   d. Suppose Jae got a score of 60% and then was allowed to retake the test. On the retake, he got all the questions right that he got right the first time, and also got half the questions right that he got wrong the first time. What is his new score?
1.4 Equations and Inequalities

Learning Objectives

- Write equations and inequalities.
- Check solutions to equations.
- Check solutions to inequalities.
- Solve real-world problems using an equation.

Introduction

In algebra, an equation is a mathematical expression that contains an equals sign. It tells us that two expressions represent the same number. For example, \( y = 12x \) is an equation. An inequality is a mathematical expression that contains inequality signs. For example, \( y \leq 12x \) is an inequality. Inequalities are used to tell us that an expression is either larger or smaller than another expression. Equations and inequalities can contain both variables and constants.

Variables are usually given a letter and they are used to represent unknown values. These quantities can change because they depend on other numbers in the problem.

Constants are quantities that remain unchanged. Ordinary numbers like 2, \( -3 \), \( \frac{3}{4} \), and \( \pi \) are constants.

Equations and inequalities are used as a shorthand notation for situations that involve numerical data. They are very useful because most problems require several steps to arrive at a solution, and it becomes tedious to repeatedly write out the situation in words.

Write Equations and Inequalities

Here are some examples of equations:

\[
3x - 2 = 5 \quad x + 9 = 2x + 5 \quad \frac{x}{3} = 15 \quad x^2 + 1 = 10
\]

To write an inequality, we use the following symbols:

> greater than

\( \geq \) greater than or equal to

< less than

\( \leq \) less than or equal to

\( \neq \) not equal to

Here are some examples of inequalities:

\[
3x - 2 < 5 \quad x + 9 \geq 2x + 5 \quad \frac{x}{3} \leq 15 \quad x^2 + 1 \neq 10
\]
The most important skill in algebra is the ability to translate a word problem into the correct equation or inequality so you can find the solution easily. The first two steps are **defining the variables** and **translating** the word problem into a mathematical equation.

**Defining the variables** means that we assign letters to any unknown quantities in the problem.

**Translating** means that we change the word expression into a mathematical expression containing variables and mathematical operations with an equal sign or an inequality sign.

**Example 1**

*Define the variables and translate the following expressions into equations.*

a) A number plus 12 is 20.

b) 9 less than twice a number is 33.

c) $20 was one quarter of the money spent on the pizza.

**Solution**

a) **Define**

Let \( n \) = the number we are seeking.

**Translate**

A number plus 12 is 20.

\[
\begin{align*}
n + 12 &= 20
\end{align*}
\]

b) **Define**

Let \( n \) = the number we are seeking.

**Translate**

9 less than twice a number is 33.

This means that twice the number, minus 9, is 33.

\[
\begin{align*}
2n - 9 &= 33
\end{align*}
\]

c) **Define**

Let \( m \) = the money spent on the pizza.

**Translate**

$20 was one quarter of the money spent on the pizza.

\[
\begin{align*}
20 &= \frac{1}{4}m
\end{align*}
\]

Often word problems need to be reworded before you can write an equation.
Example 2

Find the solution to the following problems.

a) Shyam worked for two hours and packed 24 boxes. How much time did he spend on packing one box?
b) After a 20% discount, a book costs $12. How much was the book before the discount?

Solution

a) Define

Let \( t = \) time it takes to pack one box.

Translate

Shyam worked for two hours and packed 24 boxes. This means that two hours is 24 times the time it takes to pack one box.

\[
2 = 24t
\]

Solve

\[
t = \frac{2}{24} = \frac{1}{12} \text{ hours}
\]

\[
\frac{1}{12} \times 60 \text{ minutes} = 5 \text{ minutes}
\]

Answer

Shyam takes 5 minutes to pack a box.

b) Define

Let \( p = \) the price of the book before the discount.

Translate

After a 20% discount, the book costs $12. This means that the price minus 20% of the price is $12.

\[
p - 0.20p = 12
\]

Solve

\[
p - 0.20p = 0.8p, \text{ so } 0.8p = 12
\]

\[
p = \frac{12}{0.8} = 15
\]

Answer

The price of the book before the discount was $15.

Check

If the original price was $15, then the book was discounted by 20% of $15, or $3. $15 - 3 = $12. The answer checks out.
Example 3
Define the variables and translate the following expressions into inequalities.

a) The sum of 5 and a number is less than or equal to 2.
b) The distance from San Diego to Los Angeles is less than 150 miles.
c) Diego needs to earn more than an 82 on his test to receive a $B$ in his algebra class.
d) A child needs to be 42 inches or more to go on the roller coaster.

Solution
a) Define
Let $n =$ the unknown number.
Translate

$$5 + n \leq 2$$

b) Define
Let $d =$ the distance from San Diego to Los Angeles in miles.
Translate

$$d < 150$$

c) Define
Let $x =$ Diego’s test grade.
Translate

$$x > 82$$

d) Define
Let $h =$ the height of child in inches.
Translate:

$$h \geq 42$$

Check Solutions to Equations
You will often need to check solutions to equations in order to check your work. In a math class, checking that you arrived at the correct solution is very good practice. We check the solution to an equation by replacing the variable in an equation with the value of the solution. A solution should result in a true statement when plugged into the equation.

Example 4
Check that the given number is a solution to the corresponding equation.

a) \( y = -1; \ 3y + 5 = -2y \)
b) \( z = 3; \ z^2 + 2z = 8 \)
c) \( x = -\frac{1}{2}; \ 3x + 1 = x \)

Solution

Replace the variable in each equation with the given value.

a)

\[
3(-1) + 5 = -2(-1) \\
-3 + 5 = 2 \\
2 = 2
\]

This is a true statement. This means that \( y = -1 \) is a solution to \( 3y + 5 = -2y \).

b)

\[
3^2 + 2(3) = 8 \\
9 + 6 = 8 \\
15 = 8
\]

This is not a true statement. This means that \( z = 3 \) is not a solution to \( z^2 + 2z = 8 \).

c)

\[
3 \left( -\frac{1}{2} \right) + 1 = -\frac{1}{2} \\
\left( -\frac{3}{2} \right) + 1 = -\frac{1}{2} \\
-\frac{1}{2} = -\frac{1}{2}
\]

This is a true statement. This means that \( x = -\frac{1}{2} \) is a solution to \( 3x + 1 = x \).

---

Check Solutions to Inequalities

To check the solution to an inequality, we replace the variable in the inequality with the value of the solution. A solution to an inequality produces a true statement when substituted into the inequality.

Example 5

Check that the given number is a solution to the corresponding inequality.

a) \( a = 10; \ 20a \leq 250 \)
b) \( b = -0.5; \ \frac{3-b}{b} > -4 \)
c) \( x = \frac{3}{4}; \ 4x + 5 \leq 8 \)

Solution

Replace the variable in each inequality with the given value.
a) 

\[
20(10) \leq 250 \\
200 \leq 250
\]

This statement is true. This means that \(a = 10\) is a solution to the inequality \(20a \leq 250\).

Note that \(a = 10\) is not the only solution to this inequality. If we divide both sides of the inequality by 20, we can write it as \(a \leq 12.5\). This means that any number less than or equal to 12.5 is also a solution to the inequality.

b) 

\[
\frac{3 - (-0.5)}{(-0.5)} > -4 \\
\frac{3 + 0.5}{-0.5} > -4 \\
\frac{3.5}{-0.5} > -4 \\
-7 > -4
\]

This statement is false. This means that \(b = -0.5\) is not a solution to the inequality \(\frac{3-b}{b} > -4\).

c) 

\[
4 \left(3 \frac{3}{4}\right) + 5 \geq 8 \\
3 + 5 \geq 8 \\
8 \geq 8
\]

This statement is true. It is true because this inequality includes an equals sign; since 8 is equal to itself, it is also “greater than or equal to” itself. This means that \(x = \frac{3}{4}\) is a solution to the inequality \(4x + 5 \leq 8\).

---

**Solve Real-World Problems Using an Equation**

Let’s use what we have learned about defining variables, writing equations and writing inequalities to solve some real-world problems.

**Example 6**

Tomatoes cost $0.50 each and avocados cost $2.00 each. Anne buys six more tomatoes than avocados. Her total bill is $8. How many tomatoes and how many avocados did Anne buy?

**Solution**

**Define**

Let \(a = \) the number of avocados Anne buys.

**Translate**

Anne buys six more tomatoes than avocados. This means that \(a + 6 = \) the number of tomatoes.

Tomatoes cost $0.50 each and avocados cost $2.00 each. Her total bill is $8. This means that .50 times the number of tomatoes plus 2 times the number of avocados equals 8.
1.4. Equations and Inequalities

\[
0.5(a + 6) + 2a = 8 \\
0.5a + 0.5 \cdot 6 + 2a = 8 \\
2.5a + 3 = 8 \\
2.5a = 5 \\
a = 2
\]

Remember that \(a\) = the number of avocados, so Anne buys two avocados. The number of tomatoes is \(a + 6 = 2 + 6 = 8\).

**Answer**
Anne bought 2 avocados and 8 tomatoes.

**Check**
If Anne bought two avocados and eight tomatoes, the total cost is: \((2 \times 2) + (8 \times 0.5) = 4 + 4 = 8\). The answer checks out.

**Example 7**
To organize a picnic Peter needs at least two times as many hamburgers as hot dogs. He has 24 hot dogs. What is the possible number of hamburgers Peter has?

**Solution**

**Define**
Let \(x\) = number of hamburgers

**Translate**
Peter needs at least two times as many hamburgers as hot dogs. He has 24 hot dogs.

This means that twice the number of hot dogs is less than or equal to the number of hamburgers.

\[
2 \times 24 \leq x, \text{ or } 48 \leq x
\]

**Answer**
Peter needs at least 48 hamburgers.

**Check**
48 hamburgers is twice the number of hot dogs. So more than 48 hamburgers is more than twice the number of hot dogs. The answer checks out.

**Additional Resources**


**Review Questions**

1. Define the variables and translate the following expressions into equations.
a. Peter’s Lawn Mowing Service charges $10 per job and $0.20 per square yard. Peter earns $25 for a job.
b. Renting the ice-skating rink for a birthday party costs $200 plus $4 per person. The rental costs $324 in
total.
c. Renting a car costs $55 per day plus $0.45 per mile. The cost of the rental is $100.
d. Nadia gave Peter 4 more blocks than he already had. He already had 7 blocks.

2. Define the variables and translate the following expressions into inequalities.
   a. A bus can seat 65 passengers or fewer.
   b. The sum of two consecutive integers is less than 54.
   c. The product of a number and 3 is greater than 30.
   d. An amount of money is invested at 5% annual interest. The interest earned at the end of the year is
greater than or equal to $250.
   e. You buy hamburgers at a fast food restaurant. A hamburger costs $0.49. You have at most $3 to spend.
Write an inequality for the number of hamburgers you can buy.
   f. Mariel needs at least 7 extra credit points to improve her grade in English class. Additional book reports
are worth 2 extra credit points each. Write an inequality for the number of book reports Mariel needs to
do.

3. Check whether the given number is a solution to the corresponding equation.
   a. \(a = -3;\ 4a + 3 = -9\)
   b. \(x = \frac{4}{3};\ \frac{3}{4}x + \frac{1}{2} = \frac{3}{2}\)
   c. \(y = 2;\ 2.5y - 10.0 = -5.0\)
   d. \(z = -5;\ 2(5 - 2z) = 20 - 2(z - 1)\)

4. Check whether the given number is a solution to the corresponding inequality.
   a. \(x = 12;\ 2(x + 6) \leq 8x\)
   b. \(z = -9;\ 1.4z + 5.2 > 0.4z\)
   c. \(y = 40;\ -\frac{5}{2}y + \frac{1}{4} < -18\)
   d. \(t = 0.4;\ 80 \geq 10(3t + 2)\)

5. The cost of a Ford Focus is 27% of the price of a Lexus GS 450h. If the price of the Ford is $15000, what is
the price of the Lexus?

6. On your new job you can be paid in one of two ways. You can either be paid $1000 per month plus 6%
commission of total sales or be paid $1200 per month plus 5% commission on sales over $2000. For what
amount of sales is the first option better than the second option? Assume there are always sales over $2000.

7. A phone company offers a choice of three text-messaging plans. Plan A gives you unlimited text messages for
$10 a month; Plan B gives you 60 text messages for $5 a month and then charges you $0.05 for each additional
message; and Plan C has no monthly fee but charges you $0.10 per message.
   a. If \(m\) is the number of messages you send per month, write an expression for the monthly cost of each of
the three plans.
   b. For what values of \(m\) is Plan A cheaper than Plan B?
   c. For what values of \(m\) is Plan A cheaper than Plan C?
   d. For what values of \(m\) is Plan B cheaper than Plan C?
   e. For what values of \(m\) is Plan A the cheapest of all? (Hint: for what values is A both cheaper than B and
cheaper than C?)
   f. For what values of \(m\) is Plan B the cheapest of all? (Careful—for what values is B cheaper than A?)
   g. For what values of \(m\) is Plan C the cheapest of all?
   h. If you send 30 messages per month, which plan is cheapest?
   i. What is the cost of each of the three plans if you send 30 messages per month?
1.5 Functions as Rules and Tables

Learning Objectives

- Identify the domain and range of a function.
- Make a table for a function.
- Write a function rule.
- Represent a real-world situation with a function.

Introduction

A function is a rule for relating two or more variables. For example, the price you pay for phone service may depend on the number of minutes you talk on the phone. We would say that the cost of phone service is a function of the number of minutes you talk. Consider the following situation.

Josh goes to an amusement park where he pays $2 per ride.

There is a relationship between the number of rides Josh goes on and the total amount he spends that day: To figure out the amount he spends, we multiply the number of rides by two. This rule is an example of a function. Functions usually—but not always—are rules based on mathematical operations. You can think of a function as a box or a machine that contains a mathematical operation.

Whatever number we feed into the function box is changed by the given operation, and a new number comes out the other side of the box. When we input different values for the number of rides Josh goes on, we get different values for the amount of money he spends.

The input is called the independent variable because its value can be any number. The output is called the dependent variable because its value depends on the input value.

Functions usually contain more than one mathematical operation. Here is a situation that is slightly more complicated than the example above.

Jason goes to an amusement park where he pays $8 admission and $2 per ride.

The following function represents the total amount Jason pays. The rule for this function is "multiply the number of rides by 2 and add 8."
When we input different values for the number of rides, we arrive at different outputs (costs).

These flow diagrams are useful in visualizing what a function is. However, they are cumbersome to use in practice. In algebra, we use the following short-hand notation instead:

\[ \text{input} \rightarrow f(x) \rightarrow \text{output} \]

First, we define the variables:

\[ x = \text{the number of rides Jason goes on} \]

\[ y = \text{the total amount of money Jason spends at the amusement park.} \]

So, \( x \) represents the input and \( y \) represents the output. The notation \( f() \) represents the function or the mathematical operations we use on the input to get the output. In the last example, the cost is 2 times the number of rides plus 8. This can be written as a function:

\[ f(x) = 2x + 8 \]

In algebra, the notations \( y \) and \( f(x) \) are typically used interchangeably. Technically, though, \( f(x) \) represents the function itself and \( y \) represents the output of the function.

Identify the Domain and Range of a Function

In the last example, we saw that we can input the number of rides into the function to give us the total cost for going to the amusement park. The set of all values that we can use for the input is called the domain of the function, and the set of all values that the output could turn out to be is called the range of the function. In many situations the domain and range of a function are both simply the set of all real numbers, but this isn’t always the case. Let’s look at our amusement park example.

Example 1

Find the domain and range of the function that describes the situation:

Jason goes to an amusement park where he pays $8 admission and $2 per ride.

Solution

Here is the function that describes this situation:
In this function, \( x \) is the number of rides and \( y \) is the total cost. To find the domain of the function, we need to determine which numbers make sense to use as the input (\( x \)).

- The values have to be zero or positive, because Jason can’t go on a negative number of rides.
- The values have to be integers because, for example, Jason could not go on 2.25 rides.
- Realistically, there must be a maximum number of rides that Jason can go on because the park closes, he runs out of money, etc. However, since we aren’t given any information about what that maximum might be, we must consider that all non-negative integers are possible values regardless of how big they are.

**Answer** For this function, the domain is the set of all non-negative integers.

To find the range of the function we must determine what the values of \( y \) will be when we apply the function to the input values. The domain is the set of all non-negative integers: \( \{0, 1, 2, 3, 4, 5, 6, \ldots \} \). Next we plug these values into the function for \( x \). If we plug in 0, we get 8; if we plug in 1, we get 10; if we plug in 2, we get 12, and so on, counting by 2s each time. Possible values of \( y \) are therefore 8, 10, 12, 14, 16, 18, 20... or in other words all even integers greater than or equal to 8.

**Answer** The range of this function is the set of all even integers greater than or equal to 8.

**Example 2**

Find the domain and range of the following functions.

a) A ball is dropped from a height and it bounces up to 75% of its original height.

b) \( y = x^2 \)

**Solution**

a) Let’s define the variables:

\[ x = \text{original height} \]
\[ y = \text{bounce height} \]

A function that describes the situation is \( y = f(x) = 0.75x \). \( x \) can represent any real value greater than zero, since you can drop a ball from any height greater than zero. A little thought tells us that \( y \) can also represent any real value greater than zero.

**Answer**

The domain is the set of all real numbers greater than zero. The range is also the set of all real numbers greater than zero.

b) Since there is no word problem attached to this equation, we can assume that we can use any real number as a value of \( x \). When we square a real number, we always get a non-negative answer, so \( y \) can be any non-negative real number.

**Answer**

The domain of this function is all real numbers. The range of this function is all non-negative real numbers.

In the functions we’ve looked at so far, \( x \) is called the **independent variable** because it can be any of the values from the domain, and \( y \) is called the **dependent variable** because its value depends on \( x \). However, any letters or symbols can be used to represent the dependent and independent variables. Here are three different examples:
These expressions all represent the same function: a function where the dependent variable is three times the independent variable. Only the symbols are different. In practice, we usually pick symbols for the dependent and independent variables based on what they represent in the real world—like \( t \) for time, \( d \) for distance, \( v \) for velocity, and so on. But when the variables don’t represent anything in the real world—or even sometimes when they do—we traditionally use \( y \) for the dependent variable and \( x \) for the independent variable.

For another look at the domain of a function, see the following video, where the narrator solves a sample problem from the California Standards Test about finding the domain of an unusual function: http://www.youtube.com/watch?v=NRB6s77nx2gI.

---

**Make a Table For a Function**

A table is a very useful way of arranging the data represented by a function. We can match the input and output values and arrange them as a table. For example, the values from Example 1 above can be arranged in a table as follows:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
</tr>
</tbody>
</table>

A table lets us organize our data in a compact manner. It also provides an easy reference for looking up data, and it gives us a set of coordinate points that we can plot to create a graph of the function.

**Example 3**

*Make a table of values for the function \( f(x) = \frac{1}{x} \).* Use the following numbers for input values: -1, -0.5, -0.2, -0.1, -0.01, 0.01, 0.1, 0.2, 0.5, 1.

**Solution**

Make a table of values by filling the first row with the input values and the next row with the output values calculated using the given function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>-0.5</th>
<th>-0.2</th>
<th>-0.1</th>
<th>-0.01</th>
<th>0.01</th>
<th>0.1</th>
<th>0.2</th>
<th>0.5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = \frac{1}{x} )</td>
<td>\frac{1}{-1}</td>
<td>\frac{1}{-0.5}</td>
<td>\frac{1}{-0.2}</td>
<td>\frac{1}{-0.1}</td>
<td>\frac{1}{-0.01}</td>
<td>\frac{1}{0.01}</td>
<td>\frac{1}{0.1}</td>
<td>\frac{1}{0.2}</td>
<td>\frac{1}{0.5}</td>
<td>\frac{1}{1}</td>
</tr>
<tr>
<td>( y )</td>
<td>-1</td>
<td>-2</td>
<td>-5</td>
<td>-10</td>
<td>-100</td>
<td>100</td>
<td>10</td>
<td>5</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

When you’re given a function, you won’t usually be told what input values to use; you’ll need to decide for yourself what values to pick based on what kind of function you’re dealing with. We will discuss how to pick input values throughout this book.
Write a Function Rule

In many situations, we collect data by conducting a survey or an experiment, and then organize the data in a table of values. Most often, we want to find the function rule or formula that fits the set of values in the table, so we can use the rule to predict what could happen for values that are not in the table.

**Example 4**

*Write a function rule for the following table:*

<table>
<thead>
<tr>
<th>Number of CDs</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost in $</td>
<td>24</td>
<td>48</td>
<td>72</td>
<td>96</td>
<td>120</td>
</tr>
</tbody>
</table>

**Solution**

You pay $24 for 2 CDs, $48 for 4 CDs, $120 for 10 CDs. That means that each CD costs $12.

We can write a function rule:

\[
\text{Cost} = 12 \times (\text{number of CDs}) \quad \text{or} \quad f(x) = 12x
\]

**Example 5**

*Write a function rule for the following table:*

<table>
<thead>
<tr>
<th>( x )</th>
<th>−3</th>
<th>−2</th>
<th>−1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

**Solution**

You can see that a negative number turns into the same number, only positive, while a non-negative number stays the same. This means that the function being used here is the absolute value function: \( f(x) = |x| \).

Coming up with a function based on a set of values really is as tricky as it looks. There’s no rule that will tell you the function every time, so you just have to think of all the types of functions you know and guess which one might be a good fit, and then check if your guess is right. In this book, though, we’ll stick to writing functions for linear relationships, which are the simplest type of function.

### Represent a Real-World Situation with a Function

Let’s look at a few real-world situations that can be represented by a function.

**Example 5**

*Maya has an internet service that currently has a monthly access fee of $11.95 and a connection fee of $0.50 per hour. Represent her monthly cost as a function of connection time.*

**Solution**

**Define**

Let \( x \) = the number of hours Maya spends on the internet in one month

Let \( y \) = Maya’s monthly cost
Translate
The cost has two parts: the one-time fee of $11.95 and the per-hour charge of $0.50. So the total cost is the flat fee + the charge per hour $\times$ the number of hours.

Answer
The function is $y = f(x) = 11.95 + 0.50x$.

Example 6
Alfredo wants a deck build around his pool. The dimensions of the pool are 12 feet $\times$ 24 feet and the decking costs $3 per square foot. Write the cost of the deck as a function of the width of the deck.

Solution
Define
Let $x =$ width of the deck
Let $y =$ cost of the deck

Make a sketch and label it

Translate
You can look at the decking as being formed by several rectangles and squares. We can find the areas of all the separate pieces and add them together:

$$\text{Area} = 12x + 12x + 24x + 24x + x^2 + x^2 + x^2 + x^2 = 72x + 4x^2$$

To find the total cost, we then multiply the area by the cost per square foot ($3).

Answer

$$f(x) = 3(72x + 4x^2) = 216x + 12x^2$$

Example 7
A cell phone company sells two million phones in their first year of business. The number of phones they sell doubles each year. Write a function that gives the number of phones that are sold per year as a function of how old the company is.

Solution
Define
Let $x =$ age of company in years
Let $y =$ number of phones that are sold per year
### Review Questions

1. Identify the domain and range of the following functions.
   a. Dustin charges $10 per hour for mowing lawns.
   b. Maria charges $25 per hour for tutoring math, with a minimum charge of $15.
   c. \( f(x) = 15x - 12 \)
   d. \( f(x) = 2x^2 + 5 \)
   e. \( f(x) = \frac{1}{x} \)
   f. \( f(x) = \sqrt{x} \)

2. What is the range of the function \( y = x^2 - 5 \) when the domain is -2, -1, 0, 1, 2?
3. What is the range of the function \( y = 2x - \frac{3}{4} \) when the domain is -2.5, -1.5, 5?
4. What is the domain of the function \( y = 3x \) when the range is 9, 12, 15?
5. What is the range of the function \( y = 3x \) when the domain is 9, 12, 15?
6. Angie makes $6.50 per hour working as a cashier at the grocery store. Make a table that shows how much she earns if she works 5, 10, 15, 20, 25, or 30 hours.

7. The area of a triangle is given by the formula \( A = \frac{1}{2}bh \). If the base of the triangle measures 8 centimeters, make a table that shows the area of the triangle for heights 1, 2, 3, 4, 5, and 6 centimeters.
8. Make a table of values for the function \( f(x) = \sqrt{2}x + 3 \) for input values -1, 0, 1, 2, 3, 4, 5.
9. Write a function rule for the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>9</td>
<td>16</td>
<td>15</td>
<td>36</td>
</tr>
</tbody>
</table>

10. Write a function rule for the following table:

<table>
<thead>
<tr>
<th>Hours</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
</tr>
</tbody>
</table>

11. Write a function rule for the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>24</td>
<td>12</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>
12. Write a function that represents the number of cuts you need to cut a ribbon into \( x \) pieces.
13. Write a function that represents the number of cuts you need to divide a pizza into \( x \) slices.
14. Solomon charges a $40 flat rate plus $25 per hour to repair a leaky pipe.
   a. Write a function that represents the total fee charged as a function of hours worked.
   b. How much does Solomon earn for a 3-hour job?
   c. How much does he earn for three separate 1-hour jobs?
15. Rochelle has invested $2500 in a jewelry making kit. She makes bracelets that she can sell for $12.50 each.
   a. Write a function that shows how much money Rochelle makes from selling \( b \) bracelets.
   b. Write a function that shows how much money Rochelle has after selling \( b \) bracelets, minus her investment in the kit.
   c. How many bracelets does Rochelle need to make before she breaks even?
   d. If she buys a $50 display case for her bracelets, how many bracelets does she now need to sell to break even?
1.6 Functions as Graphs

Learning Objectives

- Graph a function from a rule or table.
- Write a function rule from a graph.
- Analyze the graph of a real world situation.
- Determine whether a relation is a function.

Introduction

We represent functions graphically by plotting points on a coordinate plane (also sometimes called the Cartesian plane). The coordinate plane is a grid formed by a horizontal number line and a vertical number line that cross at a point called the origin. The origin has this name because it is the “starting” location; every other point on the grid is described in terms of how far it is from the origin.

The horizontal number line is called the \(x\)-axis and the vertical line is called the \(y\)-axis. We can represent each value of a function as a point on the plane by representing the \(x\)-value as a distance along the \(x\)-axis and the \(y\)-value as a distance along the \(y\)-axis. For example, if the \(y\)-value of a function is 2 when the \(x\)-value is 4, we can represent this pair of values with a point that is 4 units to the right of the origin (that is, 4 units along the \(x\)-axis) and 2 units up (2 units in the \(y\)-direction).
We write the location of this point as (4, 2).

**Example 1**

*Plot the following coordinate points on the Cartesian plane.*

a) (5, 3)
b) (-2, 6)
c) (3, -4)
d) (-5, -7)

**Solution**

Here are all the coordinate points on the same plot.

Notice that we move to the right for a positive $x$—value and to the left for a negative one, just as we would on a single number line. Similarly, we move up for a positive $y$—value and down for a negative one.

The $x$— and $y$—axes divide the coordinate plane into four **quadrants**. The quadrants are numbered counter-clockwise starting from the upper right, so the plotted point for (a) is in the **first** quadrant, (b) is in the **second** quadrant, (c) is in the **fourth** quadrant, and (d) is in the **third** quadrant.

---

**Graph a Function From a Rule or Table**

If we know a rule or have a table of values that describes a function, we can draw a graph of the function. A table of values gives us coordinate points that we can plot on the Cartesian plane.

**Example 2**

*Graph the function that has the following table of values.*

<table>
<thead>
<tr>
<th>$x$</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
</tr>
</tbody>
</table>

**Solution**

The table gives us five sets of coordinate points: (-2, 6), (-1, 8), (0, 10), (1, 12), and (2, 14).

To graph the function, we plot all the coordinate points. Since we are not told the domain of the function or given a real-world context, we can just assume that the domain is the set of all real numbers. To show that the function holds
for all values in the domain, we connect the points with a smooth line (which, we understand, continues infinitely in both directions).

Example 3

*Graph the function that has the following table of values.*

<table>
<thead>
<tr>
<th>Side of square</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of square</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
</tr>
</tbody>
</table>

The table gives us five sets of coordinate points: (0, 0), (1, 1), (2, 4), (3, 9), and (4, 16).

To graph the function, we plot all the coordinate points. Since we are not told the domain of the function, we can assume that the domain is the set of all non-negative real numbers. To show that the function holds for all values in the domain, we connect the points with a smooth curve. The curve does not make sense for negative values of the independent variable, so it stops at \( x = 0 \), but it continues infinitely in the positive direction.

Example 4

*Graph the function that has the following table of values.*

<table>
<thead>
<tr>
<th>Number of balloons</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>41</td>
<td>44</td>
<td>47</td>
<td>50</td>
<td>53</td>
</tr>
</tbody>
</table>

This function represents the total cost of the balloons delivered to your house. Each balloon is $3 and the store delivers if you buy a dozen balloons or more. The delivery charge is a $5 flat fee.
Solution

The table gives us five sets of coordinate points: (12, 41), (13, 44), (14, 47), (15, 50), and (16, 53).

To graph the function, we plot all the coordinate points. Since the $x$—values represent the number of balloons for 12 balloons or more, the domain of this function is all integers greater than or equal to 12. In this problem, the points are not connected by a line or curve because it doesn’t make sense to have non-integer values of balloons.

![Graph of the function](image)

In order to draw a graph of a function given the function rule, we must first make a table of values to give us a set of points to plot. Choosing good values for the table is a skill you’ll develop throughout this course. When you pick values, here are some of the things you should keep in mind.

- Pick only values from the domain of the function.
- If the domain is the set of real numbers or a subset of the real numbers, the graph will be a continuous curve.
- If the domain is the set of integers of a subset of the integers, the graph will be a set of points not connected by a curve.
- Picking integer values is best because it makes calculations easier, but sometimes we need to pick other values to capture all the details of the function.
- Often we start with one set of values. Then after drawing the graph, we realize that we need to pick different values and redraw the graph.

**Example 5**

*Graph the following function: $f(x) = |x - 2|$*

**Solution**

Make a table of values. Pick a variety of negative and positive values for $x$. Use the function rule to find the value of $y$ for each value of $x$. Then, graph each of the coordinate points.

**Table 1.8:**

| $x$  | $y = f(x) = |x - 2|$ |
|------|---------------------|
| -4   | $|-4 - 2| = -6 = 6$  |
| -3   | $|-3 - 2| = -5 = 5$  |
| -2   | $|-2 - 2| = -4 = 4$  |
| -1   | $|-1 - 2| = -3 = 3$  |
| 0    | $|0 - 2| = -2 = 2$   |
| 1    | $|1 - 2| = -1 = 1$   |
| 2    | $|2 - 2| = 0 = 0$    |
| 3    | $|3 - 2| = 1 = 1$    |
| 4    | $|4 - 2| = 2 = 2$    |
1.6. Functions as Graphs

| $x$   | $y = f(x) = |x - 2|$ |
|-------|------------------|
| 5     | $|5 - 2| = 3 = 3$  |
| 6     | $|6 - 2| = 4 = 4$  |
| 7     | $|7 - 2| = 5 = 5$  |
| 8     | $|8 - 2| = 6 = 6$  |

It is wise to work with a lot of values when you begin graphing. As you learn about different types of functions, you will start to only need a few points in the table of values to create an accurate graph.

**Example 6**

*Graph the following function: $f(x) = \sqrt{x}$*

**Solution**

Make a table of values. We know $x$ can’t be negative because we can’t take the square root of a negative number. The domain is all positive real numbers, so we pick a variety of positive integer values for $x$. Use the function rule to find the value of $y$ for each value of $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = f(x) = \sqrt{x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\sqrt{0} = 0$</td>
</tr>
<tr>
<td>1</td>
<td>$\sqrt{1} = 1$</td>
</tr>
<tr>
<td>2</td>
<td>$\sqrt{2} \approx 1.41$</td>
</tr>
<tr>
<td>3</td>
<td>$\sqrt{3} \approx 1.73$</td>
</tr>
<tr>
<td>4</td>
<td>$\sqrt{4} = 2$</td>
</tr>
<tr>
<td>5</td>
<td>$\sqrt{5} \approx 2.24$</td>
</tr>
<tr>
<td>6</td>
<td>$\sqrt{6} \approx 2.45$</td>
</tr>
<tr>
<td>7</td>
<td>$\sqrt{7} \approx 2.65$</td>
</tr>
<tr>
<td>8</td>
<td>$\sqrt{8} \approx 2.83$</td>
</tr>
<tr>
<td>9</td>
<td>$\sqrt{9} = 3$</td>
</tr>
</tbody>
</table>
Note that the range is all positive real numbers.

**Example 7**

The post office charges 41 cents to send a letter that is one ounce or less and an extra 17 cents for each additional ounce or fraction of an ounce. This rate applies to letters up to 3.5 ounces.

**Solution**

Make a table of values. We can’t use negative numbers for \( x \) because it doesn’t make sense to have negative weight. We pick a variety of positive values for \( x \), making sure to include some decimal values because prices can be decimals too. Then we use the function rule to find the value of \( y \) for each value of \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>1.2</th>
<th>1.5</th>
<th>1.8</th>
<th>2</th>
<th>2.2</th>
<th>2.5</th>
<th>2.8</th>
<th>3</th>
<th>3.2</th>
<th>3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>41</td>
<td>41</td>
<td>41</td>
<td>58</td>
<td>58</td>
<td>58</td>
<td>58</td>
<td>75</td>
<td>75</td>
<td>75</td>
<td>92</td>
<td>92</td>
</tr>
</tbody>
</table>

---

**Write a Function Rule from a Graph**

Sometimes you’ll need to find the equation or rule of a function by looking at the graph of the function. Points that are on the graph can give you values of dependent and independent variables that are related to each other by the function rule. However, you must make sure that the rule works for all the points on the curve. In this course you will learn to recognize different kinds of functions and discover the rules for all of them. For now we’ll look at some simple examples and find patterns that will help us figure out how the dependent and independent variables are related.
Example 8

The graph to the right shows the distance that an ant covers over time. Find the function rule that shows how distance and time are related to each other.

Solution

Let’s make a table of values of several coordinate points to see if we can spot how they are related to each other.

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>Distance (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4.5</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>7.5</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

We can see that for every second the distance increases by 1.5 feet. We can write the function rule as

Distance = 1.5 \times \text{time}

The equation of the function is \( f(x) = 1.5x \).

Example 9

Find the function rule that describes the function shown in the graph.

Solution

Again, we can make a table of values of several coordinate points to identify how they are related to each other.
Notice that the values of $y$ are half of perfect squares: 8 is half of 16 (which is 4 squared), 4.5 is half of 9 (which is 3 squared), and so on. So the equation of the function is $f(x) = \frac{1}{2}x^2$.

**Example 10**

*Find the function rule that shows the volume of a balloon at different times, based on the following graph:*

(Notice that the graph shows negative time. The negative time can represent what happened on days before you started measuring the volume.)

**Solution**

Once again, we make a table to spot the pattern:

<table>
<thead>
<tr>
<th>Time</th>
<th>−1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume</td>
<td>10</td>
<td>5</td>
<td>2.5</td>
<td>1.2</td>
<td>0.6</td>
<td>0.3</td>
<td>0.15</td>
</tr>
</tbody>
</table>

We can see that every day, the volume of the balloon is half what it was the previous day. On day 0, the volume is 5; on day 1, the volume is $5 \times \frac{1}{2}$; on day 2, it is $5 \times \frac{1}{2} \times \frac{1}{2}$, and in general, on day $x$ it is $5 \times \left(\frac{1}{2}\right)^x$. The equation of the function is $f(x) = 5 \times \left(\frac{1}{2}\right)^x$.

---

**Determine Whether a Relation is a Function**

A function is a special kind of *relation*. In a function, for each input there is exactly one output; in a relation, there can be more than one output for a given input.

Consider the relation that shows the heights of all students in a class. The domain is the set of people in the class and the range is the set of heights. This relation is a function because each person has exactly one height. If any person had more than one height, the relation would not be a function.
Notice that even though the same person can’t have more than one height, it’s okay for more than one person to have the same height. In a function, more than one input can have the same output, as long as more than one output never comes from the same input.

**Example 11**

*Determine if the relation is a function.*

a) \((1, 3), (-1, -2), (3, 5), (2, 5), (3, 4)\)

b) \((-3, 20), (-5, 25), (-1, 5), (7, 12), (9, 2)\)

c) 

\[
\begin{array}{cccc}
  x & 2 & 1 & 0 & 1 & 2 \\
  y & 12 & 10 & 8 & 6 & 4 \\
\end{array}
\]

**Solution**

The easiest way to figure out if a relation is a function is to look at all the \(x\)–values in the list or the table. If a value of \(x\) appears more than once, and it’s paired up with different \(y\)–values, then the relation is not a function.

a) You can see that in this relation there are two different \(y\)–values paired with the \(x\)–value of 3. This means that this relation is **not** a function.

b) Each value of \(x\) has exactly one \(y\)–value. The relation is a function.

c) In this relation there are two different \(y\)–values paired with the \(x\)–value of 2 and two \(y\)–values paired with the \(x\)–value of 1. The relation is **not** a function.

When a relation is represented graphically, we can determine if it is a function by using the **vertical line test**. If you can draw a vertical line that crosses the graph in more than one place, then the relation is not a function. Here are some examples.
Not a function. It fails the vertical line test.

A function. No vertical line will cross more than one point on the graph.

A function. No vertical line will cross more than one point on the graph.
Not a function. It fails the vertical line test.

**Additional Resources**

Once you’ve had some practice graphing functions by hand, you may want to use a graphing calculator to make graphing easier. If you don’t have one, you can also use the applet at [http://rechneronline.de/function-graphs/](http://rechneronline.de/function-graphs/). Just type a function in the blank and press Enter. You can use the options under Display Properties to zoom in or pan around to different parts of the graph.

**Review Questions**

1. Plot the coordinate points on the Cartesian plane.
   a. (4, -4)
   b. (2, 7)
   c. (-3, -5)
   d. (6, 3)
   e. (-4, 3)

2. Give the coordinates for each point in this Cartesian plane.

3. Graph the function that has the following table of values.
   a.

<table>
<thead>
<tr>
<th>x</th>
<th>-10</th>
<th>-5</th>
<th>0</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-3</td>
<td>-0.5</td>
<td>2</td>
<td>4.5</td>
<td>7</td>
</tr>
</tbody>
</table>
THIS PAGE INTENTIONALLY LEFT BLANK
7. The students at a local high school took The Youth Risk Behavior Survey. The graph below shows the percentage of high school students who reported that they were current smokers. (A current smoker is anyone who has smoked one or more cigarettes in the past 30 days.) What percentage of high-school students were current smokers in the following years?
   a. 1991
   b. 1996
   c. 2004
   d. 2005

8. The graph below shows the average life-span of people based on the year in which they were born. This information comes from the National Vital Statistics Report from the Center for Disease Control. What is the average life-span of a person born in the following years?
   a. 1940
   b. 1955
   c. 1980
   d. 1995

9. The graph below shows the median income of an individual based on his/her number of years of education. The top curve shows the median income for males and the bottom curve shows the median income for females. (Source: US Census, 2003.) What is the median income of a male that has the following years of education?
   a. 10 years of education
   b. 17 years of education

10. What is the median income of a female that has the same years of education?
    a. 10 years of education
    b. 17 years of education
11. Use the vertical line test to determine whether each relation is a function.
1.7 Problem-Solving Plan

Learning Objectives

- Read and understand given problem situations.
- Make a plan to solve the problem.
- Solve the problem and check the results.
- Compare alternative approaches to solving the problem.
- Solve real-world problems using a plan.

Introduction

We always think of mathematics as the subject in school where we solve lots of problems. Problem solving is necessary in all aspects of life. Buying a house, renting a car, or figuring out which is the better sale are just a few examples of situations where people use problem-solving techniques. In this book, you will learn different strategies and approaches to solving problems. In this section, we will introduce a problem-solving plan that will be useful throughout this book.

Read and Understand a Given Problem Situation

The first step to solving a word problem is to read and understand the problem. Here are a few questions that you should be asking yourself:

- What am I trying to find out?
- What information have I been given?
- Have I ever solved a similar problem?

This is also a good time to define any variables. When you identify your knowns and unknowns, it is often useful to assign them a letter to make notation and calculations easier.

Make a Plan to Solve the Problem

The next step in the problem-solving plan is to develop a strategy. How can the information you know assist you in figuring out the unknowns?

Here are some common strategies that you will learn:

- Drawing a diagram.
- Making a table.
• Looking for a pattern.
• Using guess and check.
• Working backwards.
• Using a formula.
• Reading and making graphs.
• Writing equations.
• Using linear models.
• Using dimensional analysis.
• Using the right type of function for the situation.

In most problems, you will use a combination of strategies. For example, looking for patterns is a good strategy for most problems, and making a table and drawing a graph are often used together. The “writing an equation” strategy is the one you will work with the most in your study of algebra.

Solve the Problem and Check the Results

Once you develop a plan, you can implement it and solve the problem, carrying out all operations to arrive at the answer you are seeking.

The last step in solving any problem should always be to check and interpret the answer. Ask yourself:

• Does the answer make sense?
• If you plug the answer back into the problem, do all the numbers work out?
• Can you get the same answer through another method?

Compare Alternative Approaches to Solving the Problem

Sometimes one specific method is best for solving a problem. Most problems, however, can be solved by using several different strategies. When you are familiar with all of the problem-solving strategies, it is up to you to choose the methods that you are most comfortable with and that make sense to you. In this book, we will often use more than one method to solve a problem, so we can demonstrate the strengths and weakness of different strategies for solving different types of problems.

Whichever strategy you are using, you should always implement the problem-solving plan when you are solving word problems. Here is a summary of the problem-solving plan.

Step 1:
Understand the problem
Read the problem carefully. Once the problem is read, list all the components and data that are involved. This is where you will be assigning your variables.

Step 2:
Devise a plan - Translate
Come up with a way to solve the problem. Set up an equation, draw a diagram, make a chart or construct a table as a start to solve your problem solving plan.

Step 3:
Carry out the plan - Solve
This is where you solve the equation you developed in Step 2.

Step 4:
Look - Check and Interpret
Check to see if you used all your information. Then look to see if the answer makes sense.

---

**Solve Real-World Problems Using a Plan**

Let’s now apply this problem solving plan to a problem.

**Example 1**

A coffee maker is on sale at 50% off the regular ticket price. On the “Sunday Super Sale” the same coffee maker is on sale at an additional 40% off. If the final price is $21, what was the original price of the coffee maker?

**Solution**

**Step 1:** Understand

We know: A coffee maker is discounted 50% and then 40%. The final price is $21.

We want: The original price of the coffee maker.

**Step 2:** Strategy

Let’s look at the given information and try to find the relationship between the information we know and the information we are trying to find.

50% off the original price means that the sale price is half of the original or $0.5 \times \text{original price}.$

So, the first sale price $= 0.5 \times \text{original price}$

A savings of 40% off the new price means you pay 60% of the new price, or $0.6 \times \text{new price}$.

$0.6 \times (0.5 \times \text{original price}) = 0.3 \times \text{original price}$ is the price after the second discount.

We know that after two discounts, the final price is $21.

So $0.3 \times \text{original price} = 21$.

**Step 3:** Solve

Since $0.3 \times \text{original price} = 21$, we can find the original price by dividing $21$ by $0.3$.

Original price $= 21 \div 0.3 = 70$.

The original price of the coffee maker was $70$.

**Step 4:** Check

We found that the original price of the coffee maker is $70$.

To check that this is correct, let’s apply the discounts.

50% of $70 = 0.5 \times 70 = 35$ savings. So the price after the first discount is original price – savings or $70 – 35 = 35$.

Then 40% of that is $0.4 \times 35 = 14$. So after the second discount, the price is $35 – 14 = 21$.

The answer checks out.
Additional Resources

The problem-solving plan used here is based on the ideas of George Pólya, who describes his useful problem-solving strategies in more detail in the book *How to Solve It*. Some of the techniques in the book can also be found on Wikipedia, in the entry [http://en.wikipedia.org/wiki/How_to_Solve_It](http://en.wikipedia.org/wiki/How_to_Solve_It).

Review Questions

1. A sweatshirt costs $35. Find the total cost if the sales tax is 7.75%.
2. This year you got a 5% raise. If your new salary is $45,000, what was your salary before the raise?
3. Mariana deposits $500 in a savings account that pays 3% simple interest per year. How much will be in her account after three years?
4. It costs $250 to carpet a room that is 14 ft by 18 ft. How much does it cost to carpet a room that is 9 ft by 10 ft?
5. A department store has a 15% discount for employees. Suppose an employee has a coupon worth $10 off any item and she wants to buy a $65 purse. What is the final cost of the purse if the employee discount is applied before the coupon is subtracted?
6. To host a dance at a hotel you must pay $250 plus $20 per guest. How much money would you have to pay for 25 guests?
7. Yusef’s phone plan costs $10 a month plus $0.05 per minute. If his phone bill for last month was $25.80, how many minutes did he spend on the phone?
8. It costs $12 to get into the San Diego County Fair and $1.50 per ride.
   a. If Rena spent $24 in total, how many rides did she go on?
   b. How much would she have spent in total if she had gone on five more rides?
9. An ice cream shop sells a small cone for $2.95, a medium cone for $3.50, and a large cone for $4.25. Last Saturday, the shop sold 22 small cones, 26 medium cones and 15 large cones. How much money did the store earn?
10. In Lise’s chemistry class, there are two midterm exams, each worth 30% of her total grade, and a final exam worth 40%. If Lise scores 90% on both midterms and 80% on the final exam, what is her overall score in the class?
11. The sum of the angles in a triangle is 180 degrees. If the second angle is twice the size of the first angle and the third angle is three times the size of the first angle, what are the measures of the angles in the triangle?
12. A television that normally costs $120 goes on sale for 20% off. What is the new price?
13. A cake recipe calls for 1 \( \frac{3}{4} \) cup of flour. Jeremy wants to make four cakes. How many cups of flour will he need?
14. Casey is twice as old as Marietta, who is two years younger than Jake. If Casey is 14, how old is Jake?
15. Kylie is mowing lawns to earn money for a new bike. After mowing four lawns, she still needs $40 more to pay for the bike. After mowing three more lawns, she has $5 more than she needs to pay for the bike.
   a. How much does she earn per lawn?
   b. What is the cost of the bike?
16. Jared goes trick-or-treating with his brother and sister. At the first house they stop at, they collect three pieces of candy each; at the next three houses, they collect two pieces of candy each. Then they split up and go down different blocks, where Jared collects 12 pieces of candy and his brother and sister collect 14 each.
   a. How many pieces of candy does Jared end up with?
   b. How many pieces of candy do all three of them together end up with?
17. Marco’s daughter Elena has four boxes of toy blocks, with 50 blocks in each one. One day she dumps them all out on the floor, and some of them get lost. When Marco tries to put them away again, he ends up with 45 blocks in one box, 53 in another, 46 in a third, and 51 in the fourth. How many blocks are missing?

18. A certain hour-long TV show usually includes 16 minutes of commercials. If the season finale is two and a half hours long, how many minutes of commercials should it include to keep the same ratio of commercial time to show time?

19. Karen and Chase bet on a baseball game: if the home team wins, Karen owes Chase fifty cents for every run scored by both teams, and Chase owes Karen the same amount if the visiting team wins. The game runs nine innings, and the home team scores one run in every odd-numbered inning, while the visiting team scores two runs in the third inning and two in the sixth. Who owes whom how much?

20. Kelly, Chris, and Morgan are playing a card game. In this game, the first player to empty their hand scores points for all the cards left in the other players’ hands as follows: aces are worth one point, face cards ten points, and all other cards are face value. When Kelly empties her hand, Morgan is holding two aces, a king, and a three; Chris is holding a five, a seven, and a queen. How many points does Kelly score?

21. A local club rents out a social hall to host an event. The hall rents for $350, and they hope to make back the rental price by charging $15 admission per person. How many people need to attend for the club to break even?

22. You plan to host a barbecue, and you expect 10 friends, 8 neighbors, and 7 relatives to show up.
   a. If you expect each person (including yourself) to eat about two ounces of potato salad, how many half-pound containers of potato salad should you buy?
   b. If hot dogs come in ten-packs that cost $4.80 apiece and hot dog buns come in eight-packs that cost $2.80 apiece, how much will you need to spend to have hot dogs and buns for everyone?
Learning Objectives

- Read and understand given problem situations.
- Develop and use the strategy “make a table.”
- Develop and use the strategy “look for a pattern.”
- Plan and compare alternative approaches to solving a problem.
- Solve real-world problems using the above strategies as part of a plan.

Introduction

In this section, we will apply the problem-solving plan you learned about in the last section to solve several real-world problems. You will learn how to develop and use the methods make a table and look for a pattern.

Read and Understand Given Problem Situations

The most difficult parts of problem-solving are most often the first two steps in our problem-solving plan. You need to read the problem and make sure you understand what you are being asked. Once you understand the problem, you can devise a strategy to solve it.

Let’s apply the first two steps to the following problem.

Example 1:

Six friends are buying pizza together and they are planning to split the check equally. After the pizza was ordered, one of the friends had to leave suddenly, before the pizza arrived. Everyone left had to pay $1 extra as a result. How much was the total bill?

Solution

Understand

We want to find how much the pizza cost.

We know that five people had to pay an extra $1 each when one of the original six friends had to leave.

Strategy

We can start by making a list of possible amounts for the total bill.

We divide the amount by six and then by five. The total divided by five should equal $1 more than the total divided by six.

Look for any patterns in the numbers that might lead you to the correct answer.

In the rest of this section you will learn how to make a table or look for a pattern to figure out a solution for this type of problem. After you finish reading the rest of the section, you can finish solving this problem for homework.
Develop and Use the Strategy: Make a Table

The method “Make a Table” is helpful when solving problems involving numerical relationships. When data is organized in a table, it is easier to recognize patterns and relationships between numbers. Let’s apply this strategy to the following example.

Example 2

Josie takes up jogging. On the first week she jogs for 10 minutes per day, on the second week she jogs for 12 minutes per day. Each week, she wants to increase her jogging time by 2 minutes per day. If she jogs six days each week, what will be her total jogging time on the sixth week?

Solution

Understand

We know in the first week Josie jogs 10 minutes per day for six days.
We know in the second week Josie jogs 12 minutes per day for six days.
Each week, she increases her jogging time by 2 minutes per day and she jogs 6 days per week.
We want to find her total jogging time in week six.

Strategy

A good strategy is to list the data we have been given in a table and use the information we have been given to find new information.

We are told that Josie jogs 10 minutes per day for six days in the first week and 12 minutes per day for six days in the second week. We can enter this information in a table:

<table>
<thead>
<tr>
<th>Week</th>
<th>Minutes per Day</th>
<th>Minutes per Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>72</td>
</tr>
</tbody>
</table>

You are told that each week Josie increases her jogging time by 2 minutes per day and jogs 6 times per week. We can use this information to continue filling in the table until we get to week six.

<table>
<thead>
<tr>
<th>Week</th>
<th>Minutes per Day</th>
<th>Minutes per Week</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>72</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
<td>84</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>96</td>
</tr>
<tr>
<td>5</td>
<td>18</td>
<td>108</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>120</td>
</tr>
</tbody>
</table>

Apply strategy/solve

To get the answer we read the entry for week six.

Answer: In week six Josie jogs a total of 120 minutes.

Check
Josie increases her jogging time by two minutes per day. She jogs six days per week. This means that she increases her jogging time by 12 minutes per week.

Josie starts at 60 minutes per week and she increases by 12 minutes per week for five weeks.

That means the total jogging time is $60 + 12 \times 5 = 120$ minutes.

**The answer checks out.**

You can see that making a table helped us organize and clarify the information we were given, and helped guide us in the next steps of the problem. We solved this problem solely by making a table; in many situations, we would combine this strategy with others to get a solution.

---

**Develop and Use the Strategy: Look for a Pattern**

Looking for a pattern is another strategy that you can use to solve problems. The goal is to look for items or numbers that are repeated or a series of events that repeat. The following problem can be solved by finding a pattern.

**Example 3**

You arrange tennis balls in triangular shapes as shown. How many balls will there be in a triangle that has 8 rows?

**Solution**

**Understand**

We know that we arrange tennis balls in triangles as shown.

We want to know how many balls there are in a triangle that has 8 rows.

**Strategy**

A good strategy is to make a table and list how many balls are in triangles of different rows.

**One row:** It is simple to see that a triangle with one row has only one ball.

**Two rows:** For a triangle with two rows, we add the balls from the top row to the balls from the bottom row. It is useful to make a sketch of the separate rows in the triangle.

$$3 = 1 + 2$$
Three rows: We add the balls from the top triangle to the balls from the bottom row.

\[ 6 = 3 + 3 \]

Now we can fill in the first three rows of a table.

**Table 1.12:**

<table>
<thead>
<tr>
<th>Number of Rows</th>
<th>Number of Balls</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

We can see a pattern.

*To create the next triangle, we add a new bottom row to the existing triangle.*

*The new bottom row has the same number of balls as there are rows.* *(For example, a triangle with 3 rows has 3 balls in the bottom row.)*

*To get the total number of balls for the new triangle, we add the number of balls in the old triangle to the number of balls in the new bottom row.*

**Apply strategy/solve:**

We can complete the table by following the pattern we discovered.

Number of balls = number of balls in previous triangle + number of rows in the new triangle

**Table 1.13:**

<table>
<thead>
<tr>
<th>Number of Rows</th>
<th>Number of Balls</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>(6 + 4 = 10)</td>
</tr>
<tr>
<td>5</td>
<td>(10 + 5 = 15)</td>
</tr>
<tr>
<td>6</td>
<td>(15 + 6 = 21)</td>
</tr>
<tr>
<td>7</td>
<td>(21 + 7 = 28)</td>
</tr>
<tr>
<td>8</td>
<td>(28 + 8 = 36)</td>
</tr>
</tbody>
</table>

**Answer** There are 36 balls in a triangle arrangement with 8 rows.

**Check**

Each row of the triangle has one more ball than the previous one. In a triangle with 8 rows,
row 1 has 1 ball, row 2 has 2 balls, row 3 has 3 balls, row 4 has 4 balls, row 5 has 5 balls, row 6 has 6 balls, row 7 has 7 balls, row 8 has 8 balls.
When we add these we get: \(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36\) balls

The answer checks out.

Notice that in this example we made tables and drew diagrams to help us organize our information and find a pattern. Using several methods together is a very common practice and is very useful in solving word problems.

**Plan and Compare Alternative Approaches to Solving Problems**

In this section, we will compare the methods of “Making a Table” and “Looking for a Pattern” by using each method in turn to solve a problem.

**Example 4**

Andrew cashes a $180 check and wants the money in $10 and $20 bills. The bank teller gives him 12 bills. How many of each kind of bill does he receive?

**Solution**

**Method 1: Making a Table**

**Understand**

Andrew gives the bank teller a $180 check.

The bank teller gives Andrew 12 bills. These bills are a mix of $10 bills and $20 bills.

We want to know how many of each kind of bill Andrew receives.

**Strategy**

Let’s start by making a table of the different ways Andrew can have twelve bills in tens and twenties.

Andrew could have twelve $10 bills and zero $20 bills, or eleven $10 bills and one $20 bill, and so on.

We can calculate the total amount of money for each case.

**Apply strategy/solve**

<table>
<thead>
<tr>
<th>$10 bills</th>
<th>$20 bills</th>
<th>Total amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0</td>
<td>$10(12) + $20(0) = $120</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>$10(11) + $20(1) = $130</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>$10(10) + $20(2) = $140</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>$10(9) + $20(3) = $150</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>$10(8) + $20(4) = $160</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>$10(7) + $20(5) = $170</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>$10(6) + $20(6) = $180</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>$10(5) + $20(7) = $190</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>$10(4) + $20(8) = $200</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>$10(3) + $20(9) = $210</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>$10(2) + $20(10) = $220</td>
</tr>
<tr>
<td>1</td>
<td>11</td>
<td>$10(1) + $20(11) = $230</td>
</tr>
<tr>
<td>0</td>
<td>12</td>
<td>$10(0) + $20(12) = $240</td>
</tr>
</tbody>
</table>

In the table we listed all the possible ways you can get twelve $10 bills and $20 bills and the total amount of money for each possibility. The correct amount is given when Andrew has six $10 bills and six $20 bills.
**Answer:** Andrew gets six $10 bills and six $20 bills.

**Check**

Six $10 bills and six $20 bills → 6($10) + 6($20) = $60 + $120 = $180

The answer checks out.

Let’s solve the same problem using the method “Look for a Pattern.”

**Method 2: Looking for a Pattern**

**Understand**

Andrew gives the bank teller a $180 check.

The bank teller gives Andrew 12 bills. These bills are a mix of $10 bills and $20 bills.

We want to know how many of each kind of bill Andrew receives.

**Strategy**

Let’s start by making a table just as we did above. However, this time we will look for patterns in the table that can be used to find the solution.

**Apply strategy/solve**

Let’s fill in the rows of the table until we see a pattern.

<table>
<thead>
<tr>
<th>$10 bills</th>
<th>$20 bills</th>
<th>Total amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0</td>
<td>$10(12) + $20(0) = $120</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>$10(11) + $20(1) = $130</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>$10(10) + $20(2) = $140</td>
</tr>
</tbody>
</table>

We see that every time we reduce the number of $10 bills by one and increase the number of $20 bills by one, the total amount increases by $10. The last entry in the table gives a total amount of $140, so we have $40 to go until we reach our goal. This means that we should reduce the number of $10 bills by four and increase the number of $20 bills by four. That would give us six $10 bills and six $20 bills.

6($10) + 6($20) = $60 + $120 = $180

**Answer:** Andrew gets six $10 bills and six $20 bills.

**Check**

Six $10 bills and six $20 bills → 6($10) + 6($20) = $60 + $120 = $180

The answer checks out.

You can see that the second method we used for solving the problem was less tedious. In the first method, we listed all the possible options and found the answer we were seeking. In the second method, we started by listing the options, but we found a pattern that helped us find the solution faster. The methods of “Making a Table” and “Looking for a Pattern” are both more powerful if used alongside other problem-solving methods.
Example 5

Anne is making a box without a lid. She starts with a 20 in. square piece of cardboard and cuts out four equal squares from each corner of the cardboard as shown. She then folds the sides of the box and glues the edges together. How big does she need to cut the corner squares in order to make the box with the biggest volume?

Solution

Step 1:

Understand

Anne makes a box out of a 20 in. × 20 in. piece of cardboard.

She cuts out four equal squares from the corners of the cardboard.

She folds the sides and glues them to make a box.

How big should the cut-out squares be to make the box with the biggest volume?

Step 2:

Strategy

We need to remember the formula for the volume of a box.

Volume = Area of base × height

Volume = width × length × height

Make a table of values by picking different values for the side of the squares that we are cutting out and calculate the volume.

Step 3:

Apply strategy/solve

Let’s “make” a box by cutting out four corner squares with sides equal to 1 inch. The diagram will look like this:
You see that when we fold the sides over to make the box, the height becomes 1 inch, the width becomes 18 inches and the length becomes 18 inches.

Volume = width × length × height

Volume = 18 × 18 × 1 = 324 in³

Let’s make a table that shows the value of the box for different square sizes:

<table>
<thead>
<tr>
<th>Side of Square</th>
<th>Box Height</th>
<th>Box Width</th>
<th>Box Length</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>18</td>
<td>18</td>
<td>18 × 18 × 1 = 324</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>16</td>
<td>16</td>
<td>16 × 16 × 2 = 512</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>14</td>
<td>14</td>
<td>14 × 14 × 3 = 588</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>12</td>
<td>12</td>
<td>12 × 12 × 4 = 576</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>10</td>
<td>10</td>
<td>10 × 10 × 5 = 500</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>8</td>
<td>8</td>
<td>8 × 8 × 6 = 384</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>6 × 6 × 7 = 252</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>4</td>
<td>4</td>
<td>4 × 4 × 8 = 128</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>2</td>
<td>2</td>
<td>2 × 2 × 9 = 36</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0 × 0 × 10 = 0</td>
</tr>
</tbody>
</table>

We stop at a square of 10 inches because at this point we have cut out all of the cardboard and we can’t make a box any more. From the table we see that we can make the biggest box if we cut out squares with a side length of three inches. This gives us a volume of 588 in³.

**Answer** The box of greatest volume is made if we cut out squares with a side length of three inches.

**Step 4:**

**Check**

We see that 588 in³ is the largest volume appearing in the table. We picked integer values for the sides of the squares that we are cut out. Is it possible to get a larger value for the volume if we pick non-integer values? Since we get the largest volume for the side length equal to three inches, let’s make another table with values close to three inches that is split into smaller increments:

<table>
<thead>
<tr>
<th>Side of Square</th>
<th>Box Height</th>
<th>Box Width</th>
<th>Box Length</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>2.5</td>
<td>15</td>
<td>15</td>
<td>15 × 15 × 2.5 = 562.5</td>
</tr>
</tbody>
</table>
Table 1.17: (continued)

<table>
<thead>
<tr>
<th>Side of Square</th>
<th>Box Height</th>
<th>Box Width</th>
<th>Box Length</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.6</td>
<td>2.6</td>
<td>14.8</td>
<td>14.8</td>
<td>14.8 \times 14.8 \times 2.6 = 569.5</td>
</tr>
<tr>
<td>2.7</td>
<td>2.7</td>
<td>14.6</td>
<td>14.6</td>
<td>14.6 \times 14.6 \times 2.7 = 575.5</td>
</tr>
<tr>
<td>2.8</td>
<td>2.8</td>
<td>14.4</td>
<td>14.4</td>
<td>14.4 \times 14.4 \times 2.8 = 580.6</td>
</tr>
<tr>
<td>2.9</td>
<td>2.9</td>
<td>14.2</td>
<td>14.2</td>
<td>14.2 \times 14.2 \times 2.9 = 584.8</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>14</td>
<td>14</td>
<td>14 \times 14 \times 3 = 588</td>
</tr>
<tr>
<td>3.1</td>
<td>3.1</td>
<td>13.8</td>
<td>13.8</td>
<td>13.8 \times 13.8 \times 3.1 = 590.4</td>
</tr>
<tr>
<td>3.2</td>
<td>3.2</td>
<td>13.6</td>
<td>13.6</td>
<td>13.6 \times 13.6 \times 3.2 = 591.9</td>
</tr>
<tr>
<td>3.3</td>
<td>3.3</td>
<td>13.4</td>
<td>13.4</td>
<td>13.4 \times 13.4 \times 3.3 = 592.5</td>
</tr>
<tr>
<td>3.4</td>
<td>3.4</td>
<td>13.2</td>
<td>13.2</td>
<td>13.2 \times 13.2 \times 3.4 = 592.4</td>
</tr>
<tr>
<td>3.5</td>
<td>3.5</td>
<td>13</td>
<td>13</td>
<td>13 \times 13 \times 3.5 = 591.5</td>
</tr>
</tbody>
</table>

Notice that the largest volume is not when the side of the square is three inches, but rather when the side of the square is 3.3 inches.

Our original answer was not incorrect, but it was not as accurate as it could be. We can get an even more accurate answer if we take even smaller increments of the side length of the square. To do that, we would choose smaller measurements that are in the neighborhood of 3.3 inches.

Meanwhile, our first answer checks out if we want it rounded to zero decimal places, but a more accurate answer is 3.3 inches.

Review Questions

1. Go back and find the solution to the problem in Example 1.
2. Britt has $2.25 in nickels and dimes. If she has 40 coins in total, how many of each coin does she have?
3. Jeremy divides a 160-square-foot garden into plots that are either 10 or 12 square feet each. If there are 14 plots in all, how many plots are there of each size?
4. A pattern of squares is put together as shown. How many squares are in the 12th diagram?

5. In Harrisville, local housing laws specify how many people can live in a house or apartment: the maximum number of people allowed is twice the number of bedrooms, plus one. If Jan, Pat, and their four children want to rent a house, how many bedrooms must it have?
6. A restaurant hosts children’s birthday parties for a cost of $120 for the first six children (including the birthday child) and $30 for each additional child. If Jaden’s parents have a budget of $200 to spend on his birthday party, how many guests can Jaden invite?

7. A movie theater with 200 seats charges $8 general admission and $5 for students. If the 5:00 showing is sold out and the theater took in $1468 for that showing, how many of the seats are occupied by students?

8. Oswald is trying to cut down on drinking coffee. His goal is to cut down to 6 cups per week. If he starts with 24 cups the first week, then cuts down to 21 cups the second week and 18 cups the third week, how many weeks will it take him to reach his goal?

9. Taylor checked out a book from the library and it is now 5 days late. The late fee is 10 cents per day. How much is the fine?

10. Mikhail is filling a sack with oranges.
   a. If each orange weighs 5 ounces and the sack will hold 2 pounds, how many oranges will the sack hold before it bursts?
   b. Mikhail plans to use these oranges to make breakfast smoothies. If each smoothie requires \( \frac{3}{4} \) cup of orange juice, and each orange will yield half a cup, how many smoothies can he make?

11. Jessamyn takes out a $150 loan from an agency that charges 12% of the original loan amount in interest each week. If she takes five weeks to pay off the loan, what is the total amount (loan plus interest) she will need to pay back?

12. How many hours will a car traveling at 75 miles per hour take to catch up to a car traveling at 55 miles per hour if the slower car starts two hours before the faster car?

13. Grace starts biking at 12 miles per hour. One hour later, Dan starts biking at 15 miles per hour, following the same route. How long will it take him to catch up with Grace?

14. A new theme park opens in Milford. On opening day, the park has 120 visitors; on each of the next three days, the park has 10 more visitors than the day before; and on each of the three days after that, the park has 20 more visitors than the day before.
   a. How many visitors does the park have on the seventh day?
   b. How many total visitors does the park have all week?

15. Lemuel wants to enclose a rectangular plot of land with a fence. He has 24 feet of fencing. What is the largest possible area that he could enclose with the fence?

16. Quizzes in Keiko’s history class are worth 20 points each. Keiko scored 15 and 18 points on her last two quizzes. What score does she need on her third quiz to get an average score of 17 on all three?

17. Tickets to an event go on sale for $20 six weeks before the event, and go up in price by $5 each week. What is the price of tickets one week before the event?

18. Mark is three years older than Janet, and the sum of their ages is 15. How old are Mark and Janet?

19. In a one-on-one basketball game, Jane scored 1 \( \frac{1}{2} \) times as many points as Russell. If the two of them together scored 10 points, how many points did Jane score?

20. Scientists are tracking two pods of whales during their migratory season. On the first day of June, one pod is 120 miles north of a certain group of islands, and every day thereafter it gets 15 miles closer to the islands. The second pod starts out 160 miles east of the islands on June 3, and heads toward the islands at a rate of 20 miles a day.
   a. Which pod will arrive at the islands first, and on what day?
   b. How long after that will it take the other pod to reach the islands?
   c. Suppose the pod that reaches the islands first immediately heads south from the islands at a rate of 15 miles a day, and the pod that gets there second also heads south from there at a rate of 25 miles a day. On what day will the second pod catch up with the first?
   d. How far will both pods be from the islands on that day?
Texas Instruments Resources

In the CK-12 Texas Instruments Algebra I FlexBook® resource, there are graphing calculator activities designed to supplement the objectives for some of the lessons in this chapter. See http://www.ck12.org/flexr/chapter/9611.
1.9 References

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